Deliberating about Voting Dimensions

(Extended Abstract)

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ABSTRACT
It has been claimed that deliberation is capable of overcoming social choice theory impossibility results, by bringing about single-peakedness. Our aim is to better understand the relationship between single-peakedness and collective justifications of preferences.

Categories and Subject Descriptors
I.2.11[Artificial Intelligence]: Distributed Artificial Intelligence—Multiagent Systems; I.4 [Social and Behavioral Sciences]: Economics

General Terms
Theory, Economics

Keywords
Judgment Aggregation, Single-peakedness, Deliberation

1. INTRODUCTION
Arrow’s theorem shows that it is not possible to aggregate individual preferences by means of an aggregation procedure that balances fairness and efficiency. Among the well-known escape routes to Arrow’s result, Black’s restriction of possible preferences to single-peaked profiles [2, 1] is significant because it has been associated with a convincing intuitive interpretation: it amounts to assuming that individuals agree on a common dimension that structures the decision problem at issue. This is an important point: any restriction of individual preferences has to be justified, as it is somehow contradicting the rationale of an aggregative view of democratic decisions: individual preferences are not matter of normative judgement, it is the aggregation procedure that carries the burden of the normative justification of choices. By contrast, deliberative democracy [4] stresses the role of public justifications of policies rather than the conditions on the aggregation of preferences. Deliberation is a discursive situation among rational and equal agents and what in principle makes a collective choice fair lies in collective justification that deliberation can bring about. In the last thirty odd years, the connection between deliberative and aggregational models have been investigated and discussed within the normative theory of democracy [4, 3]. An interesting problem is to understand to what extent the two models are compatible. A proposal of integration relies on the idea of a deliberation that is capable of promoting agents’ awareness of the relevant dimensions involved in a decision problem and that may bring about a collective justification of the elected policy [3, 7]. It is worth noting that there is a gap between the intuitive notion of a dimension and the formal condition of single-peakedness, that merely states a structural property of preference profiles. Following [3], we shall distinguish between a formal notion of a dimension, the formal dimension, that is the one in the formal definition of single-peakedness, and a semantic notion of a dimension, a semantic dimension, that is the criterion that agents use to make their choices. In this paper, we view semantic dimensions as public justifications of preferences and we discuss how formal dimensions are related to semantic dimensions. Moreover, we discuss whether single-peakedness may provide justifications of collective choices.

2. SINGLE-PEAKEDNESS
Let N be a set of agents and A a set of alternatives. For i ∈ N, a (strict) preference ordering >i is an irreflexive, transitive and complete relation >i ⊆ A × A. A preference profile P is a list of preference orderings (>1, . . . , >n). Let L(A) denote the set of all preference orderings. A social welfare function F : L(A)n → L(A) maps preference profiles to preference orderings. E.g. the majority rule is defined as F(P) = {(x, y) s.t. |{i ∈ N | (x, y) ∈ >i}| > n/2}. Single-peakedness is defined as follows. Given a linear order >, we say that y is between x and z iff x > y > z or z > y > x. The peak of preference order PEAK(>) is the maximal element wrt >. A preference profile is single-peaked if and only if there exists a linear order >1 of the alternatives (a formal dimension) such that for every >1, every alternative y such that y ̸= PEAK(>1), i prefers any alternative that is between PEAK(>1) and y (wrt >1) to y. Let >1 denote the opposite dimension. If there is an odd number of n + 1 voters and we order the agents’ peaks according to >1, the median voter’s peak, namely the option that has n/2 peaks on the right and n/2 peaks on the left is elected by majority [2].

3. EXAMPLE
We rephrase an example discussed in [6]. Suppose agents 1, 2, and 3 have to elect a collective policy among alternatives a, b and c. Their preference profile is single-peaked, e.g. wrt c > a > b (Tab. 1). Thus, there is a winning policy, i.e. a. Suppose that agents justify their preferences by appealing to three relations that express the extent to which the alternatives promote productivity P, cost C, or fairness F (Tab. 2).
E.g. 1 prefers $a$ over $b$ and 1's justification is that $a$ is more productive than $b$. We want to discuss what may provide a collective justification of the chosen policy (i.e. $a$). In order vote on justifications, agents have to agree on a common agenda. Suppose that 2 agrees to give up his justifications in terms of cost ($C$) and to use $P$ and $F$. Agent 2 can do so without revising his preferences. We obtain a profile of justifications with just $P$ and $F$ (Tab. 3). Moreover, $P$ and $F$ refer to opposite rankings, i.e. $xPy$ iff $yFx$. By voting and reasoning about the judgments in Tab. 3, we obtain the following discursive dilemma [5].

<table>
<thead>
<tr>
<th>$aFc$</th>
<th>$aFc$</th>
<th>$aPb \land aFc$</th>
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<tbody>
<tr>
<td>1</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>3</td>
<td>no</td>
<td>yes</td>
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Here, discursive dilemmas have the following interpretation. By majority, $a$ can be justified by saying that it is more productive than $b$ ($aPb$). However, $a$ is not chosen because of productivity, as it is dominated by $b$ along the productivity axis. Nor $a$ is chosen on the ground of fairness, as it is dominated by $b$ on the axis of fairness. The actual justification refers to the fact that $a$ is both more productive than $b$ and more fair than $c$. However, the dilemma shows that agents cannot elect the conjunction of the two. In the next sections, we shall generalise this example.

4. MODEL AND RESULTS

Given a preference ordering $\succ$, a justification of $\succ$ is a transitive and irreflexive relation $D$ such that $D \subseteq \succ$. A set of justifications $J = \{D_1, \ldots, D_m\}$ justifies a preference ordering $\succ$ if $J$ is a partition of $\succ$. For example, $a > b > c$ can be justified by two relations as in $J = \{\{(a, b), \}, \{(b, c), (c, a)\}\}$. Our assumptions on sets of justifications have the following meaning. Each pair of alternatives $(x, y) \succ$ is justified by some $D_i$ in $J$. Moreover, agents cannot have both $aDb$ and $bDa$. The $D_i$s are not necessarily complete, therefore some pair $(a, b)$ can be justified by $D$, whereas some other pair $(c, d)$ is justified by $D'$. For example, an agent can prefer $a$ over $b$ because “$a$ promotes GDP's growth better that $b$” and $c$ over $d$ because “$c$ is more liberal than $d$”. The transitivity of $D$s means that if an agent justifies $a \succ b$ on the ground of $D$ (e.g. “$a$ promotes fairness more than $b$”) and justifies $b \succ c$ on the same ground of $D$, then he is committed to justify also $a \succ c$ on the same ground. We assume that each agent $i$ has a set of justifications $J_i$ of $\succ$. A profile of justifications is a list $J = (J_1, \ldots, J_n)$. Single-peakedness can be easily generalized to profiles of justifications as follows: $J_i$ is compatible with a dimension $\succ_i$ iff, whenever $x \succ_i y \succ_i z$ or $x \succ_i y > \succ_i z$, and if, for $D_j \in J_i$, $x D_j y$, then for no $D_k \in J_i$, $z D_k y$.

5. CONCLUSION

Although single-peakedness provides a solution in preference aggregation, it is problematic in case of collective reasoning about justifications. We believe that this points shed some doubts on the compatibility of a deliberative model based on public justifications with aggregative models based on fair aggregation procedures, at least in case we understand agreement on collective justifications as voting.

6. REFERENCES