A LOGIC FOR REASONING ABOUT GROUP NORMS

DANIELE PORELLO
Free University of Bozen-Bolzano
daniele.porello@unibz.it

Abstract

We present a number of modal logics to reason about group norms. As a preliminary step, we discuss the ontological status of the group to which the norms are applied, by adapting the classification made by Christian List of collective attitudes into aggregated, common, and corporate attitudes. Accordingly, we shall introduce modality to capture aggregated, common, and corporate group norms. We investigate then the principles for reasoning about those types of modalities. Finally, we discuss the relationship between group norms and types of collective responsibility.

Keywords. Group norms, Group agency, Non-normal modal logics, Collective Attitudes, Deontic logic, Logics of agency, Collective responsibility.

1 Introduction

Logics for representing and reasoning about norms are very important in knowledge representation and multiagent systems as they allow for compactly express desirable properties of the agents' behaviour as well as the fine principles of interaction among agents. In this paper, we propose a logic to reason about group norms, that is, norms that apply to collectivities of individuals. In order to conceptually understand the nature of the group to which the norms are ascribed, we use the distinction made by Christian List among collective propositional attitudes [10]. We view norms as propositional attitudes, endorsing the tradition in analytic philosophy that legitimates the representation of beliefs, desires, and also norms, by means of a modal logic constructed on top of a classical propositional logic. The modalities are in fact intended to capture the mode of relationship between an agent and a propositional content, allowing us to represent the attitude of the agent with respect to the proposition. In case of deontic attitudes, the modality express the normative force holding between an agent and a state of affairs represented by a proposition.

A collective propositional attitude is, generally speaking, a propositional attitude that is ascribed to a collective entity. A map of the most salient notions of collective attitudes was proposed in [10], by distinguishing between three kinds of collective attitudes: *aggregate*, *common*, and *corporate* attitudes. Corporate attitudes presuppose that the collectivity to which the attitudes are ascribed is an *agent* in its own right, an agent who is ontologically distinguished from the mere individuals that compose the collectivity. Common attitudes are ascribed to collectivities by requiring that every member of the group share the same attitude. Common attitudes have been presupposed for instance by the debate on joint action and collective intentionality [23, 10, 12]. In this view, possible divergences among the attitudes of the members of the group are excluded. For instance, under this reading, the sentence "PC members are supposed to return the review by the deadline" is true only if every individual who is a PC member is actually committed to meet the constraint.

By contrast, aggregative attitudes do not presuppose that every member of the group share the same attitude. In this case, a propositional attitude can be ascribed to the collectivity by solving the possible disagreement by means of a voting procedure such as the majority rule. For instance, a sentence like "the parliament decided to reduce taxation" does not require that every member of the parliament actually endorses the proposal, rather, it means that a suitable winning coalition of members of the parliament votes for the proposal.

We may view the three types of collective attitudes as generating three kinds of groups that differ in the relationship between the group and its members. We shall use this distinction in order to approach a taxonomy of group norms. We shall follow in particular the analysis of group norms provided by [2, 1] where group norms are classified according to a number of parameters, such as the addressee of the norm, those that are responsible for the commitment to the norm, and those who are subject to the norm.

In this paper, we shall assume that norms constraint the *actions* of individuals or groups. For that reason, as a preliminary investigation, we shall study the principle of agency of individuals and groups by discussing the logic of agency that we may assume for those types of agents. The notion of action that we endorse in this paper is very general and abstract, as we do not want to narrow it by assuming demanding constraints nor any specific ontological view of actions. In particular, we shall place our analysis within the logic of action based on the tradition of the bringing-it-about modality [5, 8]. We introduce three logics to model the actions of groups defined in a common, aggregative, and corporate way. By means of this logics, we shall discuss the logical relationship between the actions of the group and the actions of its members. Then, we will approach a deontic logic for modelling group norms by making explicit how the collective responsibility may or may not transfer to the individuals that are members of the group.

On the technical side, the contribution of this paper is the following. We shall introduce three logics to discuss group actions that reflect the common, the aggregate, and the corporate view. To model those logics, we shall use non-normal modal logics defined by means of neighbourhood semantics, see [4] and [14] for an introduction. A number of specific principles for the modalities that express common, aggregated, and corporate actions is introduced by specifying both an Hilbert system to reason about those modalities and a semantic framework based on neighbourhood semantics to ensure soundness and completeness. Although the presentation of this paper is rather informal, we shall present the conditions on the neighbourhood functions that are required in order to prove soundness and completeness of the systems that we introduce. Finally, we shall introduce the principles of the deontic modalities that relate collective and individual responsibility, we study the Hilbert system for them, and we present the relevant semantic conditions.

The remainder of this paper is organised as follows. In Section 2, we remind the basics of non-normal or minimal modal logics and we present the logics to treat individual actions and obligations. Section 3 is dedicated to present the logics of group agency, by distinguishing common, aggregated, and corporate actions. Section 4 approaches the treatment of group norms by introducing a number of modalities for collective obligation. Section 5 concludes and indicates future work.

2 Minimal modal logics

We propose a number of logics to reason about actions and obligations of individuals and collectives. We assume a (finite) set of agents $\mathbf{N}$, and we consider the power set of $\mathbf{N}$, $2^{\mathbf{N}}$, to talk about coalitions of agents. Thus, our labels that denote agents shall range over $2^{\mathbf{N}}$. To express individual propositional attitudes, we admit singleton coalitions; in that case the meaning of a coalition C is $\{i\}$.[1]

We shall use minimal (or non-normal) modalities in order to ensure a number of basic principles to reason about agency of groups and individuals.

The semantics of the modalities that we are going to introduce is defined by mean of a *neighbourhood semantics* [4]. Let W be a set of possible wolds (or states), a neighbourhood function is a mapping $N : W \to \mathcal{P}(\mathcal{P}(W))$ that associates a world m with a set of sets of worlds (see [4, 14]). The intuitive meaning of the neighbourhood function is that it associates to each world a set of propositions that are those propositions designated to hold at w. In this setting, a neighbourhood function associates to a world w the propositions that express the available actions or the salient norm at w.

The language of propositional logic is defined as follows. Let $Prop$ be a set of propositional atoms,

$$\mathcal{L} ::= p \in Prop \mid \neg\varphi \mid \varphi \wedge \varphi$$

A valuation v is a function that associates a possible world and propositional atom to the set of truth-value $\{t, f\}$, that is $v : W \times Prop \to \{t, f\}$.

[1]This move is quite similar to the approach in [22] to discuss coalitional ability.

We define the extension of a formula φ in a model by $||\varphi|| = \{w \mid w \models \varphi\}$. The semantic definition of the modalities that we shall encounter follows the following pattern, where $\Box$ be one of the modality that we will introduce.

$$w \models \Box\varphi \text{ iff } ||\varphi|| \in N(w) \tag{1}$$

In non-normal modal logics, soundness and completeness are basically achieved by means of selecting the suitable conditions on the neighbourhood functions, see for instance [4]. In the following sections, we shall spell out the relevant conditions to achieve soundness and completeness of the proposed logics, however, for reasons of space, we shall leave a detailed proof of those results for future work.

2.1 Individual Actions and Obligations

The logic to reason about actions that we use here is based on the minimal logic of *bringing-it-about*, which was traditionally developed by [5, 8]. The principles of this logic aim to capture a very weak view of actions that, for instance, does not presuppose intentionality or explicit goals. We apply this minimal view to conceptualise collective actions of different types of groups. For instance, this weak view is adequate also for an aggregative perspective on collective actions, for which the collective is not assumed, in general, to have joint intentionality nor any shared goal, [10].

Four principles of agency are captured by the classical bringing-it-about logic [5]. The first corresponds to the axiom T of modal logics: $\mathsf{E}_i\varphi \to \varphi$, it states that if an action is brought about, then the action affects the state of the world, i.e. the formula φ that represents the effects of the action holds. The second principle corresponds to the axiom $\neg\mathsf{E}_i\top$ in classical bringing-it-about logic. It amounts to assuming that agents cannot bring about tautologies. The motivation is that a tautology is always true, regardless what an agent does, so if acting is construed as something that affects the state of the world, tautologies are not apt to be the content of something that an agent actually does. The third principle corresponds to the axiom: $\mathsf{E}_i\varphi \wedge \mathsf{E}_i\psi \to \mathsf{E}_i(\varphi \wedge \psi)$. The fourth item allows for viewing bringing it about as a modality, validating the rule of equivalents: if $\vdash \varphi \leftrightarrow \psi$ then $\vdash \mathsf{E}_i\varphi \leftrightarrow \mathsf{E}_i\psi$.

The language of the logic of bringing it about BIAT, $\mathcal{L}_{\mathsf{BIAT}}$ simply extends the language of propositional logic by adding a formula $\mathsf{E}_i\varphi$ for each individual $i \in \mathbf{N}$.

The Hilbert system for BIAT is obtained by adding the following axioms (Table 1) and the following rule to those of classical propositional logic.

$$\frac{\vdash \varphi \leftrightarrow \psi}{\vdash \mathsf{E}_i\varphi \leftrightarrow \mathsf{E}_i\psi}\ \mathsf{E}_i(\text{re})$$

- All the propositional tautologies

E1 $\mathsf{E}_i\varphi \rightarrow \varphi$

E2 $\mathsf{E}_i\varphi \wedge \mathsf{E}_i\psi \rightarrow \mathsf{E}_i(\varphi \wedge \psi)$

E3 $\neg\mathsf{E}_i\top$

Table 1: Axioms of BIAT

The semantics of BIAT is obtained by adding a number of neighbourhood functions N_i^{E}, one for each agent $i \in \mathbf{N}$. Each neighbourhood function represents the actions available to each agent at a certain world. The semantics clause for action modalities is then the following one:

$$w \models \mathsf{E}_i\varphi \text{ iff } ||\varphi|| \in N_i^{\mathsf{E}}(w) \tag{2}$$

To ensure soundness and completeness for this E, a number of conditions on the neighbourhood functions has to be ensured. For the details, we refer to [8].

2.2 Individual norms

We extend the language of classical propositional logic by adding a number of modalities for obligations O_i, for $i \in \mathbf{N}$. For the sake of simplification, we use the standard deontic logic to model individual obligations. The Hilbert system for OL extends the case of propositional logic by adding the axioms in Table 2 and by adding the following rule.

- All the propositional tautologies

O1 $\mathsf{O}_i(\varphi \rightarrow \psi) \rightarrow (\mathsf{O}_i\varphi \rightarrow \mathsf{O}_i\psi)$

O2 $\mathsf{O}_i\varphi \rightarrow \neg\mathsf{O}_i\neg\varphi$

Table 2: Axioms of OL

$$\frac{\vdash \varphi}{\vdash \mathsf{O}_i\varphi}\ \mathsf{O}_i(\text{nec})$$

Although standard deontic logic is a *normal* modal logic, we can present its semantics in terms of neighbourhood functions as well [4, 14]. Moreover, a condition on the neighbourhood function for validating O2 is required [2]

The semantics of OL can be obtained by adding a number of neighbourhood functions N_i^{O}, one for each agent $i \in \mathbf{N}$. In this case, the neighbourhood functions represent the norms that are salient for an agent at a certain world. The semantic definitions for deontic modalities are then the following one:

$$w \models \mathsf{O}_i\varphi \text{ iff } ||\varphi|| \in N_i^{\mathsf{O}}(w) \tag{3}$$

3 Group actions

We introduce three modalities for capturing a number of features of group actions, that shall be related to group norms. Here we are going to distinguish between common, aggregate, and corporate group actions, and we are going to introduce three modalities and three logics that capture their minimal principles. In particular, we highlight the principles that relates the group action with the actions of the individuals that are part of the group.

3.1 Common group actions

Common group actions are intended as those actions for which every agent of the group is indeed performing a same type of action. The axioms that govern this modality are presented in Table 3.

COM1 again reflects the effectivity of acting. COM2 and COM3 specify how to combine common actions. COM4, again, prevents tautologies to be brought about.

COM5 may in principle be questionable, as it forces the idea that the group in this case cannot do anything more that what its members jointly do. We assume it here, by endorsing a strict view of common group actions, which are in fact entirely reducible to the joint actions of the members of the group. COM6 is again questionable. For instance, suppose that every member of a parliament order a pizza, would we infer that the parliament as a group is ordering a pizza? To account for this delicate aspects, we need to separate actions that are done by individuals *qua* members of the group. We leave this points for a future dedicated work.

The rule of equivalents for common actions is expressed in the following $[\mathsf{COM}]_C$(re) rule.

---

[2]The use of non-normal modal logic to express deontic modalities was motivated in [4, 7, 18]. Moreover, non-normal deontic logics have been used to discuss institutional agency in [3] and to model weak permissions in [21]. We present here the semantic definitions in terms of neighbourhood semantics as it will be useful for simplifying the subsequent arguments.

- All the propositional tautologies

COM1 $[\mathsf{COM}]_C\varphi \to \varphi$

COM2 $[\mathsf{COM}]_C\varphi \wedge [\mathsf{COM}]_C\psi \to [\mathsf{COM}]_C(\varphi \wedge \psi)$

COM3 $[\mathsf{COM}]_C\varphi \wedge [\mathsf{COM}]_D\varphi \to [\mathsf{COM}]_{C\cup D}\varphi$

COM4 $\neg[\mathsf{COM}]_C\top$

COM5 $[\mathsf{COM}]_C\varphi \to \bigwedge_{i\in C} \mathsf{E}_i\varphi$

COM6 $\bigwedge_{i\in C} \mathsf{E}_i\varphi \to [\mathsf{COM}]_C\varphi$

Table 3: Axioms of [COM]

$$\frac{\vdash \varphi \leftrightarrow \psi}{\vdash [\mathsf{COM}]t_C\varphi \leftrightarrow [\mathsf{COM}]_C\psi}\ [\mathsf{COM}]_C(\mathrm{re})$$

The semantics of the $[\mathsf{COM}]_C$ modalities is defined as follows. For each modality, we introduce a number of neighbourhood function N_C^{COM}, one for each coalition of agents.

$$w \models [\mathsf{COM}]_C\varphi \text{ iff } ||\varphi|| \in N_C^{\mathsf{COM}}(w) \tag{4}$$

To semantically validate axioms from COM1 to COM4, the conditions are similar to those presented for the individual logics of action and for the extension to coalitions proposed by [22].

To ensure the validity of axioms COM5 and COM6, a new condition on the functions N_C^{COM} is required.

$$N_C^{\mathsf{COM}}(w) = \bigcap_{i\in C} N_i^{\mathsf{E}}(w) \text{ for every } w \in W \tag{5}$$

By means of 5, we can show that axioms COM5 and COM6 are valid as follows. For instance, we show that, for every model and every $w \in W$, $w \models [\mathsf{COM}]_C\varphi \to \bigwedge_{i\in C} \mathsf{E}_i\varphi$ (which, in fact, provides the soundness of axiom COM5 and COM6). Assume $w \models [\mathsf{COM}]_C\varphi$, then $||\varphi|| \in N_C^{COM}(w)$. Then, by condition 5, $||\varphi|| \in N_i^{\mathsf{E}}(w)$, therefore $w \models \mathsf{E}_i\varphi$.

3.2 Aggregated group actions

Aggregated actions are those that result from the outcome of an aggregation procedure, such as the majority rule, applied to the actions of the individuals. We write $[AGG]^f_C\varphi$ to express that φ is the action performed by the group C under the aggregation procedure f. An *aggregation function* is a function that maps N-tuples of 0s and 1s associated to formulas to the set $\{0,1\}$, i.e., $f : \{1,0\}^N \rightarrow \{1,0\}$. That is, f maps patterns of individual acceptance or rejections of formulas to a collective acceptance or rejection of a formula. For instance, in the simple majority rule, we assume that maj returns 1 on a majority of 1s, and it returns 0 in any other case. As a simplification move, we suppose in this paper that N is odd.

$$maj(x_1,\dots,x_N) = \begin{cases} 1 \text{ if } |\{x_i \mid x_i = 1\}| > N/2; \\ 0 \text{ otherwise} \end{cases}$$

By adjusting the acceptance threshold of $n/2$, we can define the class of uniform quota rules, where each q provides a distinct aggregation procedure.

$$quota_q(x_1,\dots,x_N) = \begin{cases} 1 \text{ if } |\{x_i \mid x_i = 1\}| > q; \\ 0 \text{ otherwise} \end{cases}$$

One may discuss the properties of such aggregators, along the lines of the traditional arguments in social choice theory and judgment aggregation [9]. For instance, the previous aggregators are *anonymous*, namely any permutation of the individual values provides the same output value. By contrast, the following two classes of aggregators, oligarchies and dictatorships, depend on specific choices of the agents.

Oligarchies: let $\{i_1,\dots,i_L\}$ be a set of indexes with $L \leq N$,
$olig(x_1,\dots,x_N) = x$ if $olig(x_{i_1},\dots,x_{i_L}) = x$.

Dictatorships of j: $d_j(x_1,\dots,x_N) = x_j$.

In the case of $olig$, the oligarchy of agents $x_{i_1},\dots,x_{i_L}$ decides the outcome; in the case of d_j, the sole agent j is decisive.

We extend the language of propositional logic by adding a number of modal operators $[\mathsf{AGG}]^f_C$ that depend on the aggregator f.

$$\mathcal{L}_{[\mathsf{AGG}]_F} ::= \varphi \in \mathcal{L} \mid [\mathsf{AGG}]^F_C\varphi$$

With this definition, as a simplification move, we are excluding possible nesting of modalities (cf. for instance [15]). By means of the aggregation function f, we can provide the semantics of the aggregated action modality as follows. Firstly, we associate to each modality $[AGG]^f_C$ a neighbourhood function N^f_C. The semantic clause is then, as usual, the following one.

$$w \models [AGG]^f_C \varphi \text{ iff } ||\varphi|| \in N^f_C(w) \quad (6)$$

Denote by $\chi_X(N^{\mathsf{E}}_i(w))$ the function that returns 1 if $X \in N^{\mathsf{E}}_i(w)$ and 0 otherwise. A winning coalition of agents wrt. an aggregator f is, informally, a set of agents that can determine the outcome. The neighbourhood functions that we consider for this logic have to satisfy the following constraint.

$$X \in N^f_C(w) \text{ iff} f(\chi_X(N_{i_1}(w)), \ldots, \chi_X(N_{i_l}(w))) = 1 \text{ for some } \{i_1, \ldots, i_l\} \subseteq C \quad (7)$$

That is, a proposition is accepted by the group C under an aggregative view that depends on the procedure f if and only if there is a winning coalition wrt. f contained in C. In particular, in case f is the majority rule, a set of words X (i.e. roughly, a proposition) is in $N^f_C(w)$ iff X is in a majority of individual neighbourhood function $N_i(w)$ contained in C.

The axiomatisation of aggregated group actions depends on the specific aggregation function that we select. For instance, in [15] an axiomatisation of the majority rule is provided. Note that it is well known from the social choice and judgment aggregation literature that the aggregation of general propositions may return inconsistent outcomes (e.g. discursive dilemmas [11].) Therefore, for preventing the logic from being inconsistent, we shall discuss which axioms of a logic of agency to drop. One solution is to drop an axiom of the form $[\mathsf{AGG}]^f_C \varphi \to \varphi$, and permit that agents in some cases may collectively accept contradictory propositions, without making the logic inconsistent. This solution applies to any aggregation procedure that may return inconsistent outcomes and permit contradictory propositions only within the scope of the $[\mathsf{AGG}]^f_C$ modalities. A second solution is to prevent inconsistent propositions to be collectively accepted, by excluding them also from the scope of the $[\mathsf{AGG}]^f_C$ modalities. This shall depend on the specific aggregation procedure and on the conditions under which it may return inconsistent sets [6]. [3]

Further principles of aggregated attitudes are left for a future dedicated work. For instance, a combination axiom such as $[\mathsf{AGG}]^f_C \varphi \wedge [\mathsf{AGG}]^f_D \psi \to [\mathsf{AGG}]^f_{C \cup D}(\varphi \wedge \psi)$ requires a careful examination of the effect of combining the outcomes of coalitions C and D wrt. f, cf. [20].

We show at least that the rule of equivalents holds for this definition of modality. Hence, aggregated group actions are legitimate modal operators.

[3] To prevent inconsistency and develop an axiom system to reason about aggregated group actions, a third solution is to use fragments of weak relevant and linear logics, cf. [16, 17]. To design logical principles that only combine collectively accepted propositions and maintain consistency, see [18].

$$\frac{\vdash \varphi \leftrightarrow \psi}{\vdash [\mathsf{AGG}]^f_C\varphi \leftrightarrow [\mathsf{AGG}]^f_C\psi}\ [\mathsf{AGG}]^f_C(\text{re})$$

Suppose that $\vdash \varphi \leftrightarrow \psi$, we have to show that, for every f, $||\varphi|| \in N^f_C(w)$ iff $\psi \in N^f_C(w)$. The assumption entails that $||\varphi|| = ||\psi||$.

We have the following chain of equivalences, which allows us to conclude:

$$||\varphi|| \in N^f_C(w) \text{ iff } f(\chi_{||\varphi||}(N_{i_1}(w)), \ldots, \chi_{||\varphi||}(N_{i_l}(w))) = 1$$
$$\text{iff}$$
$$f(\chi_{||\psi||}(N_{i_1}(w)), \ldots, \chi_{||\psi||}(N_{i_l}(w))) = 1 \text{ iff } ||\psi|| \in N^f_C(w)$$

To relate aggregative group actions to individual actions, we discuss the following two alternative assumptions. Firstly, we may view only the winning coalition of agents that were actually supporting φ as involved in the collective action resulting in φ. Secondly, we may view the entire group of agents, namely also those that were not voting for φ, as collectively bringing it about that φ. For instance, in case of a parliament passing a bill, we may view only those that voted for the bill as bringing it about, or we may view the entire parliament as acting so that the bill has passed. This distinction is reflected by selecting one between the following two axioms.

AGG1 $[\mathsf{AGG}]^f_C\varphi \to \bigwedge_{i\in C} \mathsf{E}_i\varphi$

AGG2 $[\mathsf{AGG}]^f_C\varphi \to \bigwedge_{i\in D} \mathsf{E}_i\varphi$ where D is a winning coalition wrt f.

The conditions on the neighbourhood functions that are are required in order to make AGG1 or AGG2 valid are, respectively, the following two.

$$N^f_C(w) \subseteq N^{\mathsf{E}}_i(w) \text{ for every } i \in C. \tag{8}$$

$$N^f_C(w) \subseteq N^{\mathsf{E}}_i(w) \text{ for every } i \in D, \text{ s.t. } D \text{ is a winning coalition wrt. } f. \tag{9}$$

Note that we can view aggregated actions as modalities because we decided to define aggregation procedures by means of f. This forces a property of *systematicity* on the aggregation procedure [15]. Namely, the collective acceptance only depends on the patterns of individual acceptance. In particular, any two propositions that exhibit the same pattern of acceptance and rejections are equally accepted or rejected. This condition restricts the class of aggregation functions that we are considering (e.g. by rejecting non-independent or non-neutral aggregators, see for instance [13]), but it allows us to view aggregation functions as modalities.

3.3 Corporate group actions

A corporate view of group actions requires the commitment to the existence of a *single* agent a who is the bearer of all the collective actions [10], who is in principle ontologically distinguished by the group of agents that are members of the corporate agent [19]. The group agent may be viewed as the reification of the group as a whole, distinguished from any individual of the set of agents, or it may be a specific agent who acts as a representative of the group.[4] To model this view, we enrich the set of agents $\mathbf{N}$ with a sufficient number of labels for group agents $\{a_1, \ldots, a_l\}$ and we assume that for each coalition of agents $C \in 2^{\mathbf{N}}$, there is a single group agent a_C that depends on C.

The agency of the group agent a_C is then expected to satisfy the same principles of agency of a standard individual agent. The motivation for this assumption is that the individual principles of agency are those that allow us, in this setting, to view a modality as truly agentive.

The language of corporate action modalities is defined as follows.

$$\mathcal{L}_{[\mathsf{COR}]} ::= \varphi \in \mathcal{L} \mid [\mathsf{COR}]_C\varphi \text{ where } C \in 2^{\mathbf{N}}$$

To capture the principles of corporate agents, we propose axiom [COR1], that means that the agency of the corporate agent reduces to the agency of one individual and that the agency of such an individual can be captured by the reasoning principles of the E modality. Corporate agents modalities are again assumed to satisfy the rule of equivalents.

- All the propositional tautologies

COR1 $[\mathsf{COR}]_C\varphi \to \mathsf{E}_{a_C}\varphi$ where $a_C \in \mathbf{N} \cup \{\mathbf{a_1}, \ldots, \mathbf{a_l}\}$ is a designated agent for coalition C.

Table 4: Axioms of $[\mathsf{COR}]_C$

$$\frac{\vdash \varphi \leftrightarrow \psi}{\vdash [\mathsf{COR}]_C\varphi \leftrightarrow [\mathsf{COR}]_C\psi}\ [\mathsf{COR}]_a(\text{re})$$

The semantics of corporate actions modalities is then defined as usual by introducing neighbourhood functions N_C^{COR} with the following semantic constraint.

$$w \models [\mathsf{COR}]_C\varphi \text{ iff } ||\varphi|| \in N_C^{COR}(w) \tag{10}$$

[4]Notice that a group agent is distinguished from a dictator in the sense of the dictatorial aggregation procedure of Section 3.2. For instance, the group agent may not be a member of the group C.

It is easy to see that corporate agents modalities satisfy again the rule of equivalents. To make axiom [COR1] valid, we need to assume the following constraints on the neighbourhood functions.

$$N_C^{COR}(w) \subseteq N_{a_C}^{\mathsf{E}}(w) \text{ for } a_C \in \mathbf{N} \text{ and for every } w \in W. \tag{11}$$

By means of condition (11), the rule of equivalents immediately follows.

Axiom [COR 1] is shown to be valid as follows. Suppose $w \models [\mathsf{COR}]_C\varphi$, then $||\varphi|| \in N_C^{COR}(w)$, then by condition (11), $||\varphi|| \in N_{a_C}^{\mathsf{E}}(w)$, so $w \models \mathsf{E}_{a_C}\varphi$.

We assumed the inclusion, rather then the equality, in condition 11, because we do not want to rule out the case where a_C is a standard individual agent, who acts as a representative of C in certain situations, and only part of her or his actions counts as the representative of C.

4 Group norms

We introduce three obligation modalities that relate the normative force of the collective obligation to the relevant type of action associated to the specific type of group.

The taxonomy of group norms proposed by [2] separates the dimension of agency and the dimension of type of responsibility. Here, we discussed the dimension of agency by means of the logics for [COM], [AGG], and [COR], and we approach the normative force by introducing the following modal operators $\mathsf{O}_C^{\mathsf{IR}}$, $\mathsf{O}_C^{\mathsf{RR}}$, and $\mathsf{O}_C^{\mathsf{CR}}$ to select whether the obligation induces *individual*, *representative*, or *collective* responsibility (IR, RR, or CR, respectively). We also introduce the case where the collective obligation transfers to a winning coalition of agents that can be blamed to be responsible of the collective action, we label this situation by WR. For instance, in the case of the aggregated view of collective actions, one may view as responsible of the course of action only the (winning) coalition of agents that actually supported the proposal at issue and not the whole collectivity that takes part in the decision.

We start by presenting the general principles for transferring obligations from collective agents to individuals or subgroups and then we shall discuss the interaction between types of collective obligations and collective actions. We extend the language of our logic by adding formulas of the type $\mathsf{O}_C^Y\varphi$, where $Y \in \{\mathsf{IR}, \mathsf{RR}, \mathsf{CR}, \mathsf{WR}\}$ and $C \in 2^{\mathbf{N}}$.

The axioms that capture in general how the collective obligation transfers to individuals or to subgroups are the following.

IR $\mathsf{O}_C^{\mathsf{IR}}\varphi \to \bigwedge_{i\in C} \mathsf{O}_i\varphi$

RR $\mathsf{O}_C^{\mathsf{RR}}\varphi \to \mathsf{O}_a\varphi$ where $a \in \mathbf{N} \cup \{a_1, \dots, a_l\}$.

WR $\mathsf{O}^{\mathsf{WR}}_C\varphi \to \mathsf{O}_D\varphi$ where $D \subseteq \mathbf{N}$.

The notion of collective responsibility CR is not approached here by any specific axiom: It is rather defined by the lack of transferability to any individual, representative, or winning coalitions of agents.

To provide a semantics of this new modalities, we assume a number of neighbourhood functions N^Y_C, where $Y \in \{\mathsf{IR}, \mathsf{RR}, \mathsf{CR}, \mathsf{WR}\}$ and $C \in 2^{\mathbf{N}} \cup \{a_1, \dots, a_l\}$. The truth condition of the new modal formulas is as well presented as follows:

$$w \models \mathsf{O}^Y_C\varphi \text{ iff } ||\varphi|| \in N^Y_C(w) \text{ for all } w \in W \tag{12}$$

The constraints on the neighbourhood function that are required for the validity of the relevant axioms are then the following.

$$N^{\mathsf{IR}}_C(w) \subseteq \bigcap_{i \in C} N^{\mathsf{O}}_i(w) \tag{13}$$

$$N^{\mathsf{RR}}_C(w) \subseteq N^{\mathsf{O}}_a(w) \text{ for a designated agent } a \in \mathbf{N} \cup \{a_1, \dots, a_l\} \tag{14}$$

$$N^{\mathsf{WR}}_C(w) \subseteq N^{\mathsf{O}}_D(w) \text{ for a designated } D \subseteq \mathbf{N} \tag{15}$$

It is easy to see that all this modalities satisfy the rule of equivalents:

$$\frac{\vdash \varphi \leftrightarrow \psi}{\vdash \mathsf{O}^Y_C\varphi \leftrightarrow \mathsf{O}^Y_C\psi}\ \mathsf{O}^Y_C(\text{re})$$

We show for instance that condition (15) allows for establishing the validity of Axiom WR. Assume that $w \models \mathsf{O}^{\mathsf{WR}}_C\varphi$, then $||\varphi|| \in N^{\mathsf{WR}}_C(w)$. By means of condition 15, we have that $||\varphi|| \in N^{\mathsf{O}}_D(w)$, thus $w \models O_D\varphi$.

Whether the collective obligations, as such, shall also satisfy the principles of standard deontic logic is left for future work. Here we approached obligations based on individual responsibility (IR) and corporate responsibility (CR) by reducing them to individual obligations. The case of group obligation based on winning coalitions (WR), by contrast, requires understanding the principles of group obligations along the lines of [18].

4.1 Discussion

We conclude by discussing a number of examples. In principle, we can permit every combination of the modalities $\mathsf{O}^{\mathsf{IR}}_C$, $\mathsf{O}^{\mathsf{RR}}_C$, and $\mathsf{O}^{\mathsf{CR}}_C$ with the types of group agency [COM], [AGG], and [COR], therefore expressing in this framework the taxonomy of [2]. In fact, we shall see that a few cases are delicate.

Firstly, by means of IR and of the view of common group actions, we can infer that, if the group has an obligation towards φ, then in this case every agent has an obligation towards φ.

$$\vdash \mathsf{O}^{\mathsf{IR}}_{C}[\mathsf{COM}]_C\varphi \rightarrow \mathsf{O}_i\mathsf{E}_i\varphi \tag{16}$$

We can show that (16) is valid as follows. Suppose $w \models \mathsf{O}^{\mathsf{IR}}_{C}[\mathsf{COM}]_C\varphi$, then by IR, we conclude $w \models \mathsf{O}_i[\mathsf{COM}]_C\varphi$. By Axiom $\mathsf{O}1$ and COM 5, we infer that $w \models \mathsf{O}_i \bigwedge_{i\in C} \mathsf{E}_i\varphi$ and, again by $\mathsf{O}1$, we conclude $\mathsf{O}_i\mathsf{E}_i\varphi$.

Moreover, if we view the responsibility of the group action as ascribed to a representative of the agents, say a, we can infer the following principle.

$$\vdash \mathsf{O}^{\mathsf{RR}}_{C}[\mathsf{COR}]_C\varphi \rightarrow \mathsf{O}_a\mathsf{E}_a\varphi \tag{17}$$

Assume $w \models \mathsf{O}^{\mathsf{RR}}_{C}[\mathsf{COM}]_C\varphi$. By Axiom RR, we infer that $w \models \mathsf{O}_a[\mathsf{COR}]_C\varphi$. Then, by COR1 and $\mathsf{O}1$ we conclude $\mathsf{O}_a\mathsf{E}_a\varphi$.

Consider now the following formula.

$$\vdash \mathsf{O}^{\mathsf{RR}}_{C}[\mathsf{COM}]_C\varphi \rightarrow \mathsf{O}_a\mathsf{E}_a\varphi \text{ only if } a \in C \tag{18}$$

Can we ascribe a representative responsibility to an group action defined by means of a common action? Formula (18) is derivable in our framework, only if the representative agent is among those in C. This makes sense since a common action of the group C is supposed to refer to the actions of the individuals in C. Therefore, in principle, we may allow for a representative agent of the common action, although it has to be part of the group.

For aggregated group actions, again we may select whether the responsibility is at the individual, coalitional, or representative level. For instance, the following formula is derivable only in case we assume that every individual in C is actually bringing about φ, even if she or he is voting against φ (cf. axiom AGG 1).

$$\vdash \mathsf{O}^{\mathsf{IR}}_{C}[\mathsf{AGG}]_C\varphi \rightarrow \mathsf{O}_i\mathsf{E}_i\varphi \tag{19}$$

By contrast, the following formula holds in case we assume axiom AGG 2.

$$\vdash \mathsf{O}^{\mathsf{WR}}_{C}[\mathsf{AGG}]^f_C\varphi \rightarrow \mathsf{O}_i\mathsf{E}_i\varphi \; D \subseteq \mathbf{N} \text{ winning coalition for } f \text{ and } i \in D. \tag{20}$$

To establish (20), we reason as follows. From $w \models \mathsf{O}^{\mathsf{WR}}_{C}[\mathsf{AGG}]^f_C\varphi$, then by WR, we have that $\mathsf{O}_D[\mathsf{AGG}]^f_C\varphi$, where D is a winning coalition wrt. f. By means of AGG2 and $\mathsf{O}1$, we infer that $\mathsf{O}_i\mathsf{E}_i\varphi$, for $i \in D$. In this case, i.e. by assuming AGG 2 instead of AGG 1, formula (20) fails in case the agent i is not a member of the winning coalition D.

Aggregation procedure are in fact quite versatile, as they can also be viewed as abstract representation of the decision mechanisms of an organisation. For instance, an oligarchic aggregation can in principle represent decisions taken at the level of the board of directors of a company.

5 Conclusion

We have introduced three logics to reason about common, aggregative, and corporate actions, by relating the agency of the group to the agency of the individuals that are members of the group. We have then introduced a number of deontic principles that relate collective responsibility to individual responsibility and we have discussed a few combinations of type of group action and type of responsibility. We have informally introduced those systems, however the conditions on the neighbourhood functions that we have presented are those that are required in order to establish soundness and completeness of our systems. By means of the logics that we have introduced, we can provide a logical foundation, which is also grounded in the philosophical analysis of groups developed in [10], of the taxonomy of group norms provided in [2].

Future work shall present a detailed proof of the completeness results that we suggested in this paper. Moreover, due to the simplicity of the systems that we have introduced, we conjecture that they are all decidable, future work shall establish this fact. Finally, we are interested in studying in detail the principles that relate the group action and the individual action, in particular, by expanding the analysis of aggregated group actions and of corporate group actions, which constitute the delicate cases.

References

[1] H. Aldewereld, V. Dignum, and W. Vasconcelos. We ought to; they do; blame the management! In *Coordination, Organizations, Institutions, and Norms in Agent Systems IX*, pages 195–210. Springer, 2014.

[2] H. Aldewereld, V. Dignum, and W. W. Vasconcelos. Group norms for multi-agent organisations. *ACM Transactions on Autonomous and Adaptive Systems (TAAS)*, 11(2):15, 2016.

[3] J. Carmo and O. Pacheco. Deontic and action logics for organized collective agency, modeled through institutionalized agents and roles. *Fundamenta Informaticae*, 48(2-3):129–163, 2001.

[4] B. Chellas. *Modal Logic: An Introduction*. Cambridge University Press, 1980.

[5] D. Elgesem. The modal logic of agency. *Nordic J. Philos. Logic*, 2(2), 1997.

[6] U. Endriss, U. Grandi, and D. Porello. Complexity of judgment aggregation. *Journal of Artificial Intelligence Research*, 45:481–514, 2012.

[7] L. Goble. A proposal for dealing with deontic dilemmas. In *Deontic Logic in Computer Science*, pages 74–113. Springer, 2004.

[8] G. Governatori and A. Rotolo. On the Axiomatisation of Elgesem's Logic of Agency and Ability. *Journal of Philosophical Logic*, 34:403–431, 2005.

[9] C. List. The theory of judgment aggregation: An introductory review. *Synthese*, 187(1):179–207, 2012.

[10] C. List. Three kinds of collective attitudes. *Erkenntnis*, 79(9):1601–1622, 2014.

[11] C. List and P. Pettit. Aggregating sets of judgments: An impossibility result. *Economics and Philosophy*, 18(1):89–110, 2002.

[12] C. List and P. Pettit. *Group Agency. The possibility, design, and status of corporate agents*. Oxford University Press, 2011.

[13] C. List and C. Puppe. Judgment aggregation: A survey. In *Handbook of Rational and Social Choice*. Oxford University Press, 2009.

[14] E. Pacuit. *Neighborhood Semantics for Modal Logics*. Springer, 2017.

[15] M. Pauly. Axiomatizing collective judgment sets in a minimal logical language. *Synthese*, 158(2):233–250, 2007.

[16] D. Porello. A proof-theoretical view of collective rationality. In *IJCAI 2013, Proceedings of the 23rd International Joint Conference on Artificial Intelligence, Beijing, China, August 3-9, 2013*, 2013.

[17] D. Porello. Judgement aggregation in non-classical logics. *Journal of Applied Non-Classical Logics*, 27(1-2):106–139, 2017.

[18] D. Porello. Logics for modelling collective attitudes. *Fundam. Inform.*, 158(1-3):239–275, 2018.

[19] D. Porello, E. Bottazzi, and R. Ferrario. The ontology of group agency. In *Formal Ontology in Information Systems - Proceedings of the Eighth International Conference, FOIS 2014, September, 22-25, 2014, Rio de Janeiro, Brazil*, pages 183–196, 2014.

[20] D. Porello and N. Troquard. Non-normal modalities in variants of linear logic. *Journal of Applied Non-Classical Logics*, 25(3):229–255, 2015.

[21] O. Roy, A. J. Anglberger, and N. Gratzl. The logic of obligation as weakest permission. In *Deontic Logic in Computer Science*, pages 139–150. Springer, 2012.

[22] N. Troquard. Reasoning about coalitional agency and ability in the logics of "bringing-it-about". *Autonomous Agents and Multi-Agent Systems*, 28(3):381–407, 2014.

[23] R. Tuomela. *Social ontology: Collective intentionality and group agents*. Oxford University Press, 2013.