Majoritarian Group Actions

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Abstract. In this paper, we introduce a logic to reason about group actions for groups that are defined by means of the majority rule. It is well known that majoritarian aggregation is subject to irrationality, as the results in social choice theory and judgment aggregation show. The logic of action that we use here for modelling group actions is based on a substructural propositional logic that allows for preventing inconsistent outcome. Agency is modeled by means of a "bringing-it-about" modal logic with coalitions. We show that, in this way, it is possible to obtain a consistent model of agency of groups that are defined in an aggregative manner.

1 Introduction

The rationality of group attitudes, such as beliefs, desires, intentions, and agency is a central issue in the foundation of multiagent system. The concept of group attitudes has been interpreted in different ways by a number of approaches. For instance, Christian List [10] distinguishes between three kinds of collective attitudes: *aggregate*, *common*, and *corporate* group attitudes. We are here interested in the first two kinds. Common attitudes are ascribed to a group by requiring that every member of the group share the same attitude. Common attitudes have been presupposed by the approach to group actions based on collective intentionality and joint action [26, 10, 12]. In this view, possible disagreements among the members of the group are excluded. By contrast, an aggregative view of group attitudes does not presuppose that the individuals that are members of the group all share the same attitude. A group attitude can be ascribed to the group by solving the disagreement by means of an aggregation procedure such as the majority rule. This view is appealing, since it seems to be capable of accounting for the common attitudes perspective, that implies unanimity, but also for a number of situations in which it is reasonable to define a group attitude without assuming that all the members of the group share a common attitudes. For instance, in case we model parliaments, organizations, committees. Besides being descriptively adequate to a number of modelling scenarios, non-unanimous group attitudes are important also from the point of view of multiagent systems and knowledge representation of group information. Consider the following situations involving artificial agents. Suppose three sensors have been placed in different locations of a room and they are designed to trigger a fire alarm in case they detect smoke. By viewing the three sensors as a group, we may investigate what are the conditions that defines the group action, in this case, "trigger the alarm". By forcing unanimity, that is, by viewing group attitudes as common attitudes, we are assuming that the three sensors as a group trigger the alarm only in the case they all agree in detecting smoke. However, a unanimous view of group actions may lead to lose a lot of relevant information: if the sensors disagree, the alarm is simply not triggered, even if the disagreement may be caused, for instance, by the fact that one of the three sensors is in a location that has not been reached by the smoke yet. Unanimity appears to be a too restrictive requirement in this case [20]. An aggregative view provides the formal means to tailor the concept of group information to the specific scenario, by selecting the appropriate aggregation procedure. Although an aggregative view of group attitudes is desirable for many reasons, several results in social choice theory and judgment aggregation show that many important aggregation procedures are not capable of guaranteeing a rational outcome. One crucial example is the majority rule [11]. As usual in the BDI approach to agency, at least a modicum of rationality has to be presupposed in order to define an agent. An agent cannot hold (synchronically) inconsistent attitudes, such as plans, commitments or beliefs. When the outcome of an aggregation procedure is inconsistent, as in the case of the majority rule, we simply cannot define a majoritarian group as an agent. The solution that has been developed in the literature on judgment aggregation is to give up procedures such as the majority rule and investigate aggregation procedures that guarantee consistency [13].

In this paper, we are interested in pursuing a different strategy. We want to be able to ascribe collective attitudes to groups even in the case individual attitudes are aggregated by means of the majority rule. The motivation is that many real scenarios actually use the majority rule to settle disagreement. Besides, the majority rule has a number of desirable features such as it is simple to understand and implement, preference aggregation is strategy-proof (when consistent) [3], it has been associated to an epistemic interpretation justified by the Condorcet's jury theorem.

In order to avoid the inconsistency, as well as the irrationality caused by the majority rule, we shall weaken the logic that we use to model group rationality in order to ensure general consistency. For instance, we know that the majority rule may return inconsistent sets of judgments. On a closer inspection, the inconsistency is deeply intertwined with the principles of classical logic. In [19], a possibility result for the majority rule has been provided by means of linear logic [7, 8]. We will build on that in order to develop a logic for *majoritarian groups* attitudes, that is, groups whose attitudes are aggregatively defined by means of the majority rule. In principle, our treatment can be instantiated to a number of propositional attitudes such as beliefs, desires, intentions. We focus here on action and agency and we model the situation in which a number of agents submit proposals for action that have to be aggregated in a group action.

In order to model action and agency, we shall use a "bringing-it-about" modality whose properties have been investigated for instance by [5, 9]. A logic of agency based on intuitionistic linear logic has been presented in [21]. Moreover, a insightful discussion of bringing-it-about modalities for representing coalitional ability has been done in [25]. A closely related work is [2]. There, the authors use judgment aggregation in order to model group attitudes by relying on logics of agency. The significant difference with respect to the present paper is that their treatment applies to aggregation procedures that are known to guarantee consistency, e.g. the premise based procedure [13], whereas here we are interested in approaching the majority rule.

Modelling group actions in case of an aggregative view is simpler than modelling common group actions in a number of aspects. Firstly, we do not need to assume joint intentionality nor a shared goal. By definition of majoritarian group, we are already assuming that individuals do have different goals [10]. For that reason, we are labelling the actions of the group by *group* actions and not by *joint* action. Moreover, it means that intentionality and goals do not need to enter the model for defining what a group action is [12]. That is, the "bringing-it-about" modality is sufficient for this preliminary task.

The remainder of this paper is organized as follows. In the next section, after presenting the basic notions of judgment aggregation, we shall introduce the problem of majoritarian group actions in an informal way by discussing a discursive dilemma [11]. In Section 3, we see the basics of the propositional substructural logic that we are going to use to build our model of group actions. In Section 4, we enrich the propositional substructural logic by means of the modalities for agency. Besides a number of technicalities, the difference with [21] is that we are going to introduce coalitions in the spirit of [25]. Moreover, we are going to establish soundness and completeness of the Hilbert system for our logic. In Section 5, we approach our modelling of group actions and we show how to provide a consistent modellisation of majoritarian aggregation. In particular, we show how to view discursive dilemmas as examples of the complex nature of group actions, rather than a case of mere logical inconsistency. Finally, Section 6 concludes.

2 Background on Judgment Aggregation

We present now the basic definitions of the judgement aggregation (JA) setting [13, 6], which provides the formal counterpart of an aggregative view of collective attitudes [10]. We slightly rephrase the definitions for the present application. Let N be a (finite) set of agents. An *agenda* $\mathcal{X}_{\mathcal{L}}$ is a (finite) set of propositions in the language $\mathcal{L}_L$ of a given logic L that is closed under complements. i.e. non-double negations. We let the logic L unspecified here, as we shall see two logics for defining judgment sets. Moreover, we shall assume that the agenda does not contain tautologies or contradictions, this is motivated in a number of papers in JA [13] but it is also motivated here by the fact that we are assuming that it is not meaningful to talk about tautological actions (cf. Section 4.2).

The standard definitions of the JA framework are the following. A *judgement set* J is a subset of $\mathcal{X}_L$ such that J is (wrt L) *consistent* ($J \nvdash_L \bot$), *complete* (for all $\varphi \in \mathcal{X}_L$, $\varphi \in J$ or $\neg\varphi \in J$) and *deductive closed* (if $J \vdash_L \varphi$ and $\varphi \in \mathcal{X}_L$, $\varphi \in J$). The definitions are presented in syntactic terms, by referring to a calculus $\vdash_L$. The notion of consistency has been rephrased to cope with the logic that we are going to introduce. Note that our definition are equivalent to their usual model-theoretic counterparts.

Denote by $J(\mathcal{X}_L)$ the set of all judgement sets on $\mathcal{X}_L$. A *profile* of judgements sets $\mathbf{J}$ is a vector $(J_1, \ldots, J_n)$, where $n = |N|$. An *aggregator* is then a function $F : J(\mathcal{X}_L)^n \to \mathcal{P}(\mathcal{X}_L)$. The codomain of F is the powerset $\mathcal{P}(\mathcal{X}_L)$, therefore admitting possibly inconsistent sets. Let $N_\varphi = \{i \mid \varphi \in J_i\}$, the majority rule is defined as follows: $m : J(\mathcal{X}_L)^n \to \mathcal{P}(\mathcal{X}_L)$ such that $m(\mathbf{J}) = \{\varphi \in \mathcal{X}_L \mid |N_\varphi| > n/2\}$.

In JA, the collective set $F(\mathbf{J})$ is also assumed to be consistent, complete, and deductively closed wrt. L. This defines the notion of *collective rationality*. The classical results in JA show that the majority rule is not collectively rational. That means that there exists an agenda and a profile of judgment sets such that $F(\mathbf{J})$ is not consistent. We present now a significant example.

We are endorsing an aggregative view of group attitudes, that means that the attitudes that one can ascribe the group are obtained as outcomes of an aggregation procedure [10]. Consider the following case of discursive dilemma [11] on the agenda of propositions $\{A, B, A \wedge B, \neg A, \neg B, \neg(A \wedge B)\}$.

	A	$A \wedge B$	B	$\neg A$	$\neg(A \wedge B)$	$\neg B$
1	yes	yes	yes	no	no	no
2	no	no	yes	yes	yes	no
3	yes	no	no	no	yes	yes
maj.	yes	no	yes	no	yes	no

By majority, the group accepts A, because of 1 and 3, it accepts B, because of 1 and 2, and it also has to accept $\neg(A \wedge B)$. Therefore, one can see that the group is inconsistent since, for instance, A and B entails $A \wedge B$ which contradicts $\neg(A \wedge B)$, i.e. $A \wedge B, \neg(A \wedge B) \vdash \bot$.

We are interested in representing majoritarian group reasoning and the outcomes of an election by means of the bringing-it-about modality E. For instance, suppose the group G is assumed to be the agent who is bringing about the propositions accepted by majority. To express that, we write formulas such as $E_G A$, $E_G B$, and $E_G \neg(A \wedge B)$. By means, of the usual principles of the modality E — for instance the axiom T: $\mathsf{E}_G \varphi \rightarrow \varphi$ — we can infer again the inconsistency between $A \wedge B$ and $\neg(A \wedge B)$.

We will see that a fundamental point in order to save collective rationality in case of majoritarian decisions is to keep track of the coalitions that are responsible for supporting the collectively accepted propositions [19]. In order to do that, we want to reason about formulas that indicates that the coalition$\{1, 3\}$ brings about A, coalition $\{1,2\}$ brings about B, and coalition $\{2, 3\}$ does not brings about $(A \wedge B)$. We write such statements as follows: $\mathsf{E}_{\{1,3\}} A$,$\mathsf{E}_{\{1,2\}} B$, $\mathsf{E}_{\{2,3\}} \neg(A \wedge B)$. We will see that the inconsistency in discursive dilemmas is caused by mixing propositions that hold because they are supported by a single coalition, e.g. $\mathsf{E}_{\{1,3\}} A$, and propositions that hold because they follow from propositions that are supported by two distinct coalitions: for instance, $\mathsf{E}_{\{1,2,3\}}(A \wedge B)$ that follows from $\mathsf{E}_{\{1,3\}} A$ and $\mathsf{E}_{\{1,2\}} B$.

In the next section, we will introduce a logic that is capable of distinguishing two modes of combinations of propositions supported by coalitions, preventing the majoritarian outcomes from inconsistency.

3 Background on Substructural Logics

We briefly introduce the basics of Linear Logic (LL). LL captures a resource-sensitive reasoning that means that, for instance, wrt linear logic implication $\multimap$, *modus ponens* $A, A \multimap B \vdash B$ is valid only if the right amount of assumptions is given, so that $A, A, A \multimap B \nvdash B$. This implication has been interpreted as a form of *causal* connection between the antecedent and the consequence [8]: the antecedent is *consumed* by the causal process and it is not available for further inferences. In order to achieve resource-sensitivity of the entailment, linear logic rejects the global validity of the structural rules of the sequent calculus: contraction (C) and weakening (W). Rejecting (W) amounts to preventing monotonicity of the entailment and rejecting (C) blocks the possibility of making indistinguishable copies of the assumptions. By rejecting (W) and (C), we are lead to split the classical connectives into two classes: *multiplicatives* and *additives*. For instance, the classical conjunction $\wedge$ splits into two distinct operators: the multiplicative $\otimes$ ("tensor") and the additive $\&$ ("with") [7, 8]. Since monotonicity fails in general, the tensor conjunction for instance does *not* satisfy $A \otimes B \not\multimap B$ nor $A \otimes B \not\multimap A$, by contrast the additive conjunction does: $A \mathbin{\&} B \multimap A$ and $A \mathbin{\&} B \multimap B$. Analogous distinction can be made for disjunction. We will use an intuitionistic variant of linear logic, thus we shall have the implication $\multimap$ instead of the multiplicative disjunction. By slightly abusing the notation, we will denote the additive disjunction by $\vee$.

For our purpose, the resource-sensitive nature of linear logic is fundamental as it is capable of handling an important distinction between the truth makers of a proposition: we will see that a formula $A \otimes B$ will be made true by two different coalitions of agents, one supporting A and one supporting B, whereas $A \mathbin{\&} B$ will be made true by a single coalition, cf. [19]. For the sake of simplicity, we shall stick to a fragment of intuitionistic linear logic (exponential-free). Moreover, as we shall see in the next section, we assume distributivity of additive connective $\&$ over $\vee$. Distributivity is not valid in linear logic. By slightly abusing the notation, we shall call our fragment by ILLD[1]. The motivation for adding distributivity is mainly technical: it is due to the fact that we can still prove soundness and completeness wrt a simple Kripke-like model.

The language of ILLD, $\mathcal{L}_{\mathsf{ILLD}}$, then is defined as follows:

$$A ::= \mathbf{1} \mid p \mid A \otimes A \mid A \mathbin{\&} A \mid A \multimap A \mid A \vee A$$

where $p \in Atom$.

3.1 Hilbert system for ILLD

We introduce the Hilbert system for ILLD, that has been basically developed in [1], see also [24, 16] .We define the Hilbert-style calculus by introducing a list of axioms in Table **??** and by defining the following notion of deduction. The concept of deduction of linear logic requires a tree-structure in order to handle the hypothesis in the correct resource-sensitive way. This entails that, in particular, in linear logic, every *modus ponens* application (cf. $\multimap$-rule) applies to a single occurrence of A and of $A \multimap B$.

The notion of proof in the Hilbert system is defined as follows.

Definition 1 (Deduction in H-ILLD). *A* deduction tree *in* H-ILLD $\mathcal{D}$ *is inductively constructed as follows. (i) The leaves of the tree are assumptions* $A \vdash A$*, for* $A \in \mathcal{L}_{\mathsf{ILLD}}$*, or* $\vdash B$ *where* B *is an axiom in Table 1 (base cases).*

(ii) We denote by $\Gamma \overset{\mathcal{D}}{\vdash} A$ *a deduction tree with conclusion* $\Gamma \vdash A$*. If* $\mathcal{D}$ *and* $\mathcal{D}'$ *are deduction trees, then the following are deduction trees (inductive steps).*

[1] Note that, since distributivity hold, ILL D is also known as a *contractionless relevance logic* [16], which is a decidable relevance logic [15]. We leave a proper comparison with the families of substructural and relevance logics for future work.

1. $\vdash A \multimap A$
2. $\vdash (A \multimap B) \multimap ((B \multimap C) \multimap (A \multimap C))$
3. $\vdash (A \multimap (B \multimap C)) \multimap (B \multimap (A \multimap C))$
4. $\vdash A \multimap (B \multimap A \otimes B)$
5. $\vdash (A \multimap (B \multimap C)) \multimap (A \otimes B \multimap C)$
6. $\vdash \mathbf{1}$
7. $\vdash \mathbf{1} \multimap (A \multimap A)$
8. $\vdash (A \mathbin{\&} B) \multimap A$
9. $\vdash (A \mathbin{\&} B) \multimap B$
10. $\vdash ((A \multimap B) \mathbin{\&} (A \multimap C)) \multimap (A \multimap B \mathbin{\&} C)$
11. $\vdash A \multimap A \vee B$
12. $\vdash B \multimap A \vee B$
13. $(A \multimap C) \mathbin{\&} (B \multimap C) \multimap (A \vee B \multimap C)$
14. $A \mathbin{\&} (B \vee C) \multimap (A \mathbin{\&} B) \vee (A \mathbin{\&} C)$
15. $(A \vee B) \mathbin{\&} (A \vee C) \multimap A \mathbin{\&} (B \vee C)$

Table 1. Axioms of ILL

$$\frac{\overset{\mathcal{D}}{\Gamma \vdash A} \qquad \overset{\mathcal{D}'}{\Gamma' \vdash A \multimap B}}{\Gamma, \Gamma' \vdash B} \multimap\textit{-rule} \qquad \frac{\overset{\mathcal{D}}{\Gamma \vdash A} \qquad \overset{\mathcal{D}'}{\Gamma \vdash B}}{\Gamma \vdash A \mathbin{\&} B} \mathbin{\&}\textit{-rule}$$

3.2 Models of ILL

A Kripke-like class of models for ILLD is substantially due to Urquhart [27]. A *Kripke resource frame* is a structure $\mathcal{M} = (M, e, \circ, \geq)$, where $(M, e, \circ)$ is a commutative monoid with neutral element e, and $\geq$ is a pre-order on M. The frame has to satisfy the condition of *bifunctoriality*: if $m \geq n$, and $m' \geq n'$, then $m \circ m' \geq n \circ n'$. To obtain a *Kripke resource model*, a valuation on atoms $V : Atom \to \mathcal{P}(M)$ is added. It has to satisfy the *heredity* condition: if $m \in V(p)$ and $n \geq m$ then $n \in V(p)$. The truth conditions of the formulas of $\mathcal{L}_{\mathsf{ILLD}}$ in the Kripke resource model $\mathcal{M} = (M, e, \circ, \geq, V)$ are the following:

$m \models_{\mathcal{M}} p$ iff $m \in V(p)$.
$m \models_{\mathcal{M}} \mathbf{1}$ iff $m \geq e$.
$m \models_{\mathcal{M}} A \otimes B$ iff there exist m_1 and m_2 such that $m \geq m_1 \circ m_2$ and $m_1 \models_{\mathcal{M}} A$ and $m_2 \models_{\mathcal{M}} B$.
$m \models_{\mathcal{M}} A \mathbin{\&} B$ iff $m \models_{\mathcal{M}} A$ and $m \models_{\mathcal{M}} B$.
$m \models_{\mathcal{M}} A \vee B$ iff $m \models_{\mathcal{M}} A$ or $m \models_{\mathcal{M}} B$
$m \models_{\mathcal{M}} A \multimap B$ iff for all $n \in M$, if $n \models_{\mathcal{M}} A$, then $n \circ m \models_{\mathcal{M}} B$.

Denote $||A||^{\mathcal{M}}$ the extension of A in $\mathcal{M}$, i.e. the set of worlds of $\mathcal{M}$ in which A holds. A formula A is *true* in a model $\mathcal{M}$ if $e \models_{\mathcal{M}} A$.[2] A formula A is *valid* in Kripke resource frames, noted $\models A$, iff it is true in every model. The heredity condition can be straightforwardly proved to extend naturally to every formula, that is: For every formula

[2] When the context is clear we will write $||A||$ instead of $||A||^{\mathcal{M}}$, and $m \models A$ instead of $m \models_{\mathcal{M}} A$.

A, if $m \models A$ and $m' \geq m$, then $m' \models A$. By means of this semantics, it is possible to prove that ILL D is sound and complete wrt to the class of Kripke models [27].

4 Linear Bringing-it-about Logic with coalitions (Linear BIAT C)

The (non-normal modal) logic of agency of bringing-it-about [5, 9] has been traditionally developed on top of classical propositional logic. In [21], a version of bringing-it-about based on ILL has been developed as a logic for modeling resource-sensitive actions of a single agent. In the next section, we will propose a version Linear BIAT with coalitions, based on ILLD. We simply label it Linear BIAT C. The *bringing-it-about* modality has been discussed in particular by [5, 9]. For each agent a in a set of agents $\mathcal{A}$, the modality $\mathsf{E}_a A$ specifies that agent $a \in \mathcal{A}$ brings about A. The following principles captures the intended notion of agency [5]:

1. If something is brought about, then this something holds.
2. It is not possible to bring about a tautology.
3. If an agent brings about two things concomitantly then the agent also brings about the conjunction of these two things.
4. If two statements are equivalent, then bringing about one is equivalent to bringing about the other.

The logical meaning of the four principle is the following. The first item corresponds to the axiom T of modal logics: $\mathsf{E}_i A \multimap A$. It states that bringing-it-about is effective: if an action is brought about, then the action affects the state of the world, i.e. the formula A that represents the execution of the action holds. The second item corresponds to the axiom $\neg \mathsf{E}_i \top$ (notaut) in classical bringing-it-about logic. It amounts to assuming that agents cannot bring about tautologies. The motivation is that a tautology is always true, regardless what an agent does, so if acting is construed as something that affects the state of the world, tautologies are not apt to be the content of something that an agent actually does. Item 3 corresponds to the axiom: $\mathsf{E}_i A \wedge \mathsf{E}_i B \rightarrow \mathsf{E}_i(A \wedge B)$. We shall discuss this principle in detail, when we will approach the linear version of this logic. The fourth item allows for viewing bringing it about as a modality, obeying the rule of equivalents: if $\vdash A \leftrightarrow B$ then $\vdash \mathsf{E}_i A \leftrightarrow \mathsf{E}_i B$.

4.1 Axioms of Linear BIAT C

We assume a set of coalitions $\mathbf{C}$ that is closed by disjoint union $\sqcup$. In this version of BIAT logic, agents are replaced by coalition. We admit singleton coalitions, in that case the meaning of a coalition C in $\mathbf{C}$ is $\{i\}$. This move is similar to those made in [25] to discuss coalitional ability. The language of Linear BIAT with coalition, $\mathcal{L}_{L\mathsf{BIAT}C}$ simply extends the definition of $\mathcal{L}_{\mathsf{ILL}D}$, by adding a formula $\mathsf{E}_C A$ for each coalition $C \in \mathbf{C}$. The axioms of Linear BIAT C are presented in Table 4.1. The Hilbert system is defined by extending the notion of deduction in Definition 1 by means of the new axioms in Table 4.1 and of two new rules for building deduction trees, cf. Definition 2.

A number of important differences are worth noticing, when discussing the principle of agency in linear logics. Principle 1 is captured by Axiom 16, that is, the linear version of T: $\mathsf{E}_a A \multimap A$. Since in linear logics all the tautologies are not provably equivalent, principle 2 changes into an inference rule, that is ($\sim$ nec) in Definition 2: if $\vdash A$, then $E_C A \vdash \bot$. That means that, if a formula is a theorem, a coalition that brings it about implies the contradiction[3]. Moreover, the rule ($\mathsf{E}_C$re) captures the fourth principle.

The principle for combining actions (Item 3 in the list) is crucial here: it can be interpreted in linear logic in two ways, namely, in a multiplicative way by $\otimes$ (Axiom 18) and in an additive way by $\&$ (Axiom 17) The distinction between the two types of combination is crucial for preventing collective irrationality [19]. The point is that the multiplicative combination, in our interpretation, requires two different winning coalitions that support the propositions, whereas the additive combination forces the same coalition to support both propositions. This distinction is reflected by the resource-sensitive nature of the two conjunctions. For instance, one can prove that $C \multimap A, D \multimap B \vdash C \otimes D \multimap A \otimes B$ and $C \multimap A, C \multimap B \vdash C \multimap A \,\&\, B$, that is in the former case the combination of hypotheses $B \otimes C$ is required, whereas in the latter only C is required. Therefore, Axiom 17 means that if the same coalition brings about A and brings about B, then the same coalition can bring about the combination of A and B: $A \,\&\, B$.

We define the disjoint union of two coalitions $C \sqcup D$ by $C \cup D$, if $C \cap D = \emptyset$ and $C \sqcup D = (C \times \{1\}) \cup (D \times \{0\})$, otherwise. Axiom 18 means that if a coalition C brings about action A and coalition C' brings about action B then, the disjoint union of two coalitions $C \sqcup C'$ brings about the combination of actions $A \otimes B$. It is important to stress that the condition of disjointness of C and C' is crucial for modelling the group actions defined by majority in a consistent way. In particular, the condition shows that the individuals that are member of the coalition are all equally relevant to make the proposition accepted. Take for instance the case of $\mathsf{E}_{\{1,2\}} A$ and $\mathsf{E}_{\{2,3\}}$. If we enable the inference to $\mathsf{E}_{\{1,2,3\}} A \otimes B$, then we would lose the information concerning the possibly crucial contribution of agent 2 in both coalitions.

Axiom 17 and 18 are reminiscence of Coalition Logic [17]. Note that we do not assume any further axiom of coalition logic. For instance, no coalition monotonicity. That is motivated by the fact that we are modelling *profile-reasoning*, that is, we start by a fixed profile of individual attitudes and we want to capture, by means of the modality E, how the group reasons about those propositions that have been accepted by majority in that profile. In this setting, given a profile of individual attitudes, there exists only one coalition that supports a proposition that has been accepted by majority. This is a different perspective wrt coalition logic and logic of coalitional ability [25].

Moreover, the principles for combining actions, such as Axiom 17 and 18, have been criticized on the ground that coalitions C and D may have different goals, therefore it is not meaningful to view the action of $C \sqcup D$ as a joint action. However, the aggregative view of group actions defined by means of the majority rule presupposes that the group is not defined by means of a shared goal. Therefore, Axioms 17 and 18 are legitimate from this point of view.

The following definition extends the concept of deduction to Linear BIAT C.

Definition 2 (Deduction in Linear BIAT C). *A* deduction tree *in Linear* BIAT *C denoted by $\mathcal{D}$ is inductively constructed as follows. (i) The leaves of the tree are assump-*

[3] This amounts to negating $E_C A$, according to intuitionistic negation.

- All the axioms of ILL (cf. Table 1)
16 $\mathsf{E}_C A \multimap A$
17 $\mathsf{E}_C A \,\&\, \mathsf{E}_C B \multimap \mathsf{E}_C(A \,\&\, B)$
18 $\mathsf{E}_C A \otimes \mathsf{E}_D B \multimap \mathsf{E}_{C \sqcup D}(A \otimes B)$

Table 2. Axioms of Linear BIAT

tions $A \vdash A$, for $A \in \mathcal{L}_{\mathsf{LBIAT}C}$, or $\vdash B$ where B is an axiom in Table 2 (base cases). (ii) If $\mathcal{D}$ and $\mathcal{D}'$ are deduction trees, then the trees in Definition 1 are also deduction trees in Linear BIAT. *Moreover, the following are deduction trees (inductive steps).*

$$\frac{\overset{\mathcal{D}}{\vdash A \multimap B} \qquad \overset{\mathcal{D}'}{\vdash B \multimap A}}{\vdash \mathsf{E}_C A \multimap \mathsf{E}_C B}\ \mathsf{E}_C(re) \qquad \frac{\vdash A}{\vdash \mathsf{E}_C A \multimap \bot} \sim nec$$

4.2 Models of Linear BIAT C

The semantics of the bringing-it-about modality is defined by adding a neighborhood semantics on top of the Kripke resource frame. A neighborhood function is a mapping $N : M \to \mathcal{P}(\mathcal{P}(M))$ that associates a world m with a set of sets of worlds (see [4]). The intuitive meaning of the neighborhood in this setting is that it associates to each world a set of propositions that can be done by coalition C. Neighborhood functions are related to effectivity function introduced in Social Choice Theory [14] for modelling coalitional power.

In order to interpret the modalities in a modal Kripke resource frame, we take one neighborhood function N_C for every coalition $C \in \mathbf{C}$ and we define:

$m \models \mathsf{E}_C A$ iff $||A|| \in N_C(m)$

Note that it is possible that $m \models \mathsf{E}_C A$, yet $m' \not\models \mathsf{E}_C A$ for some $m' \geq m$. That is, heredity may fail in the extension of $\models$ for $\mathcal{L}_{L\mathsf{BIAT}C}$. We will then require our neighborhood function to satisfy the condition that if some set $X \subseteq M$ is in the neighborhood of a world, then X is also in the neighborhood of all the worlds that are above according to $\geq$.

$$\text{if } X \in N_C(m) \text{ and } n \geq m \text{ then } X \in N_C(n) \tag{1}$$

The rule ($\mathsf{E}_C$re) does not require any further condition on Kripke resource frames, it is already true because of the definition of E_C.

The rule ($\sim$ nec) requires:

$$\text{if } (X \in N_C(w)) \text{ and } (e \in X) \text{ then } (w \in V(\bot)) \tag{2}$$

Axiom 16 requires:

$$\text{if } \ X \in N_C(w) \text{ then } w \in X \tag{3}$$

We turn now to action compositions. Axiom 17 requires:

$$\text{if } X \in N_C(w) \text{ and } Y \in N_C(w), \text{ then } X \cap Y \in N_C(w) \tag{4}$$

Let $X \circ Y = \{x \circ y \mid x \in X \text{ and } y \in Y\}$, the condition corresponding to the multiplicative version of action combination, Axiom 18, requires that the upper closure of $X \circ Y$, denote it by $(X \circ Y)^{\uparrow}$, is in $N_{C \sqcup D}(x \circ y)$:

$$\text{if } X \in N_C(x) \text{ and } Y \in N_D(y) \text{ , then } (X \circ Y)^{\uparrow} \in N_{C \sqcup D}(x \circ y) \tag{5}$$

Summing up, Linear BIAT is evaluated over the following models:

Definition 3. *A* modal Kripke resource model *is a structure* $\mathcal{M} = (M, e, \circ, \geq, N_C, V)$ *such that:*

- $(M, e, \circ, \geq)$ *is a Kripke resource frame;*
- *For any* $C \in \boldsymbol{C}$*,* N_C *is a neighborhood function that satisfies conditions (1), (2), (3), (4), and (5).*
- V *is a valuation on atoms,* $V : Atom \to \mathcal{P}(M)$*.*

Heredity is true as well for Linear BIAT C over modal Kripke resource models for modal formulas, as an easy induction shows.

4.3 Soundness and completeness

We approach now the proof of soundness and completeness of Linear BIAT C wrt Kripke resource frames that satisfy the conditions we put. The proof for the propositional case is mainly due to [27]. A proof of soundness and completeness for Linear BIAT in case of as single agent is provided in [21, 22]. The proof that we present here is a simple adaptation of those proofs for the case of the Hilbert system for Linear BIAT C.

Theorem 1 (Soundness of Linear BIAT with Coalitions). *Linear BIAT C is sound wrt the class of Kripke resource frames that satisfy (1) (2), (3), (4), and (5): if* $\Gamma \vdash A$*, then* $\Gamma \models A$*.*

Proof. We only present the cases for axioms 17 and 18. The other cases are handled in similar way in [21].

We show that axiom 17 is valid. That is, for every model, $e \models \mathsf{E}_C A \,\&\, \mathsf{E}_C B \multimap \mathsf{E}_C(A \,\&\, B)$. That means, by definition of $\multimap$, for every x, if $x \models \mathsf{E}_C A \,\&\, \mathsf{E}_C B$, then $x \models \mathsf{E}_C(A \,\&\, B)$. If $x \models \mathsf{E}_C A \,\&\, \mathsf{E}_C B$, then $x \models \mathsf{E}_C A$ and $x \models \mathsf{E}_C B$, that entails, by definition of E_C, that $||A|| \in N_C(x)$ and $|B|| \in N_C(x)$. Thus, by condition (4), we infer $||A|| \cap ||B|| \in N_C(x)$. That means $x \models \mathsf{E}_C(A \,\&\, B)$.

We show that axiom 18 is valid, $e \models \mathsf{E}_C A \otimes \mathsf{E}_D B \multimap \mathsf{E}_{C \sqcup D}(A \otimes B)$. That is, for every x, if $x \models \mathsf{E}_C A \otimes \mathsf{E}_D B$, then $x \models \mathsf{E}_{C \sqcup D}(A \otimes B)$. If $x \models \mathsf{E}_C A \otimes \mathsf{E}_D B$, then by definition of $\otimes$, there exist y and z, such that $x \geq y \circ z$ and $y \models \mathsf{E}_C A$ and $z \models \mathsf{E}_D B$. Therefore, $||A|| \in N_C(y)$ and $||B|| \in N_D(z)$, this by condition (5), we infer that $(||A|| \circ ||B||)^{\uparrow} \in N_{C \sqcup D}(y \circ z)$. Thus, since $x \geq y \circ z$, by condition (5), $(||A|| \circ ||B||)^{\uparrow} \in N_{C \sqcup D}(x)$, that is $x \models \mathsf{E}_{C \sqcup D}(A \otimes B)$.

We turn now to show completeness. Firstly, we define the canonical model, which is adapted from [21].

In the following, $\sqcup_m$ is the multiset union. Also, we denote by $\Delta^\star = A_1 \otimes \cdots \otimes A_m$, for $A_i \in \Delta$. Moreover, the extension of A in the canonical model is $|A|^c = \{\Gamma \mid \Gamma \vdash A\}$.

Definition 4. *Let $\mathcal{M}^c = (M^c, e^c, o^c, \geq^c, N^c, V^c)$ such that:*

- *$M^c = \{\Gamma \mid \Gamma$ is a finite multiset of formulas$\}$;*
- *$\Gamma \circ^c \Delta = \Gamma \sqcup_m \Delta$;*
- *$e^c = \emptyset$;*
- *$\Gamma \geq^c \Delta$ iff $\Gamma \vdash \Delta^*$;*
- *$\Gamma \in V^c(p)$ iff $\Gamma \vdash p$;*
- *For every $C \in \boldsymbol{C}$, $N^c_C(\Gamma) = \{\mid A \mid^c \mid \Gamma \vdash \mathsf{E}_C A\}$.*

Lemma 1. *$\mathcal{M}^c$ is a modal Kripke resource model that satisfies (1) (2), (3), (4), and (5).*

Proof. We only show the case of condition (4), and (5), which differs from the proof in [21].

Case of Condition (4). Suppose $X \in N^c_C(\Gamma)$ and $Y \in N^c_C(\Gamma)$. By definition of N^c_C, $X \in \{X =\mid A \mid^c \mid \Gamma \vdash \mathsf{E}_C A\}$, thus $\Gamma \vdash \mathsf{E}_C A$ is provable in the Hilbert system. Analogously, $\Gamma \vdash \mathsf{E}_C B$, where $Y = |B|^c$. Then, we can prove in the Hilbert system that $\Gamma \vdash \mathsf{E}_C A \,\&\, \mathsf{E}_C B$, by means of the &-rule:

$$\dfrac{\overset{\mathcal{D}}{\Gamma \vdash \mathsf{E}_C A} \qquad \overset{\mathcal{D}'}{\Gamma \vdash \mathsf{E}_C B}}{\Gamma \vdash \mathsf{E}_C A \,\&\, \mathsf{E}_C B}\ \&\text{-rule}$$

By axiom 12 and $\multimap$-rule (i.e. *modus ponens*), we conclude $\Gamma \multimap \mathsf{E}_C(A \,\&\, B)$ as follows:

$$\dfrac{\Gamma \vdash \mathsf{E}_C A \,\&\, \mathsf{E}_C B \qquad \vdash \mathsf{E}_C A \,\&\, \mathsf{E}_C B \multimap \mathsf{E}_C(A \,\&\, B)}{\Gamma \vdash \mathsf{E}_C A \,\&\, B}\ \multimap\text{-rule}$$

Since $\Gamma \vdash \mathsf{E}_c A \,\&\, B$, we have that $||A \,\&\, B|| \in N^c_C(\Gamma)$. Therefore, we can conclude since $||A \,\&\, B|| = ||A|| \cap ||B|| = X \cap Y$.

Case of Condition (5). Assume $X \in N^c_C(\Gamma)$, $Y \in N^c_D(\Delta)$. By definition of canonical neighborhood, we have: $\Gamma \vdash \mathsf{E}_C A$, $\Delta \vdash \mathsf{E}_D B$, where $||A|| = X$ and $||B|| = Y$. We can prove that $\Gamma, \Delta \vdash \mathsf{E}_C A \otimes \mathsf{E}_D B$ as follows.

$$\multimap\text{-rule}\ \dfrac{\dfrac{\Gamma \vdash \mathsf{E}_C A \qquad \vdash \mathsf{E}_C A \multimap (\mathsf{E}_D B \multimap (\mathsf{E}_C A \otimes \mathsf{E}_D B))\ \text{(ax. 4)}}{\Gamma \vdash \mathsf{E}_D B \multimap \mathsf{E}_C A \otimes \mathsf{E}_D B}\ \multimap\text{-rule} \qquad \Delta \vdash \mathsf{E}_D B}{\Gamma, \Delta \vdash \mathsf{E}_C A \otimes \mathsf{E}_C B}$$

By means of axiom 18, we infer $\Gamma, \Delta \vdash \mathsf{E}_{C \sqcup D}(A \otimes B)$.

$$\frac{\Gamma, \Delta \vdash \mathsf{E}_C A \otimes \mathsf{E}_C B \qquad \vdash \mathsf{E}_C A \otimes \mathsf{E}_D B \multimap \mathsf{E}_{C \cup D}(A \otimes B) \text{ (ax 13, } C \cap D = \emptyset)}{\Gamma, \Delta \vdash \mathsf{E}_{C \cup D} A \otimes B} \multimap\text{-rule}$$

Therefore, $(||A|| \circ ||B||)^{\uparrow} \in N^c_{C \sqcup D}(\Gamma \circ \Delta)$. We conclude by noticing that $(X \circ Y)^{\uparrow} = (||A|| \circ ||B||)^{\uparrow}$.

We are ready now to prove the truth lemma. The proof is as usual by induction on the complexity of the formula A and there is no significative difference wrt the proof in [21]. We denote by $\Gamma \models^c A$ the satisfaction relation wrt the canonical model.

Lemma 2 (Truth lemma). *If $\Gamma \models^c A$, then $\Gamma \vdash A$.*

As usual, by means of the truth lemma, one establishes completeness.

Theorem 2 (Completeness of Linear BIAT with Coalitions). *Linear BIAT C is sound wrt the class of Kripke resource frames that satisfy (1) (2), (3), (4), and (5): If $\Gamma \models A$, then $\Gamma \vdash A$.*

5 Aggregative view of group attitudes

We want to interpret the relationship between individual and collective attitudes by means of the logic Linear BIAT C. However, the majority rule is not interpreted within the logic by means of a logic formula, as for instance in [23, chapter 4]. Recall that in intiuitionistic logics, one can define $\sim A = A \multimap \bot$.

We want to associate to each individual judgment set J_i, that contains formulas of an agenda defined in classical logic, a set $\bar{J}_i$ of E_i-formulas of Linear BIAT C. Recall that the *additive connectives* of Linear BIAT C are $\&$ and $\vee$ and the *multiplicative connectives* are $\otimes$ and $\multimap$.

If φ is a formula in classical logic, then its *additive translation* in Linear BIAT C is defined as follows: $p' = p$, for p atomic; $(A \wedge B)' = A' \,\&\, B'$ and $(A \vee B)' = A' \vee B'$.

For each individual judgment set J_i, we define the set $\bar{J}_i = \{\mathsf{E}_i \varphi' \mid \varphi \in J_i\}$. That is, we view the elements of the agenda that are supported by an agent i as actions that she/he is proposing to bringing about as a group action. Moreover, it is easy to see that, if J_i is a judgment set (i.e. it is individually rational) according to classical logic, then $\bar{J}_i$ is a judgment set (individually rational) wrt to Linear BIAT C.

Note that any $\bar{J}_i$ cannot contain multiplicative formulas. Firstly, by the additive translation, any φ occurring in $\mathsf{E}_i\varphi$ is additive (i.e. it contains only $\&$, $\vee$, $\sim$). Secondly, $\bar{J}_i$ cannot infer any multiplicative formula of the form $\mathsf{E}_i\varphi$, since that the only axiom that would entail $\mathsf{E}_i\varphi$ where φ is a multiplicative formula is Axiom 18, but that demands making the disjoint union of coalitions, e.g. from $\mathsf{E}_i A$, $\mathsf{E}_i B$ one can only infer $\mathsf{E}_{i \sqcup i}(A \otimes B)$.

This motivates the role of the additive translation $()'$ in modelling individual attitudes: by means of Linear BIAT C, we can view individual judgment sets as supported by a single coalition, that is, the coalition made by the agent i who is supporting her/his propositions. Therefore, multiplicative formulas cannot be in the individual judgment sets, because they would require the attitudes of at least another agent.

We associate now a set of formulas in Linear BIAT C to the set of formulas obtained by majority $m(\mathbf{J})$ for a given profile $\mathbf{J}$. Denote such a set by J_G. We say that coalition C *supports* φ in profile $\mathbf{J}$ iff $C = N_\varphi$. Thus,

$$\bar{J}_G = \{\mathsf{E}_C\varphi \mid \varphi \in m(\mathbf{J}) \text{ and } C \text{ supports } \varphi \text{ in } \mathbf{J}\} \quad (6)$$

For instance, in the example of discursive dilemma in Section 2, we have the following sets of formulas of Linear BIAT C:

$\bar{J}_1 = \{\mathsf{E}_1 A, \mathsf{E}_1 B, \mathsf{E}_1(A \,\&\, B)\}$
$\bar{J}_2 = \{\mathsf{E}_2 \sim A, \mathsf{E}_2 B, \mathsf{E}_2 \sim (A \,\&\, B)\}$
$\bar{J}_3 = \{\mathsf{E}_3 A, \mathsf{E}_3 \sim B, \mathsf{E}_3 \sim (A \,\&\, B)\}$
$\bar{J}_G = \{\mathsf{E}_{1,3} A, \mathsf{E}_{1,2} B, \mathsf{E}_{2,3} \sim (A \,\&\, B)\}$

Note that each set $\bar{J}_i$ is consistent and complete wrt Linear BIAT C.[4] We show that $\bar{J}_G$ is consistent, complete and deductively closed wrt Linear BIAT C.

Definition 5 (Majoritarian group reasoning). *Majoritarian group reasoning is defined as the deductive closure wrt Linear* BIAT *C of $\bar{J}_G$: $cl(\bar{J}_G)$.*

By adapting the proof in [19], we can show that group reasoning by means of Linear BIAT C is always consistent, that is, for every profile of judgment sets, although $m(\mathbf{J})$ may be inconsistent wrt classical logic, $\bar{J}_G$ is consistent wrt Linear BIAT C.

Theorem 3. *For every profile* $\mathbf{J}$*, majoritarian group reasoning $cl_(\bar{J}_G)$ is consistent, complete, and deductively closed wrt Linear* BIAT *C.*

Proof. In [19], it is proved that, for every agenda of formulas defined in the language of additive linear logic, the majority rule is always consistent, i.e. for any profile $\mathbf{J}$, $m(\mathbf{J})$ is consistent. The proof is based on the fact that, in additive linear logic, every minimally inconsistent[5] set has cardinality 2. This follows from the fact that every deduction in the additive fragment of linear logic contains exactly two formulas $A \vdash B$, therefore every minimally inconsistent set must be of the form $A, \sim B \vdash$.

By means of the characterization in [13, 6], one can infer that, if every minimally inconsistent subset of an agenda $\mathcal{X}$ has cardinality less then 3, then the majority rule is consistent for every profile of judgment sets defined on $\mathcal{X}$.

Thus, firstly, we need to show that every minimally inconsistent set in additive linear logic *plus* distributivity (axiom 14 and 15) has cardinality 2. We show that this is the case. If every deduction in the additive fragment of Linear BIAT contains exactly two formulas, then this holds also for the additive fragment plus axioms 14 and 15. It is sufficient to notice that, if $\Gamma \vdash A$ is derivable in the additive fragment, since axioms 14 and 15 are of the form $\varphi_1 \multimap \varphi_2$, by means of them and of the $\multimap$-rule the number of formulas in the derivation does not increase.

[4] Note that $\bar{J}$ is consistent iff it is not the case that $\bar{J} \multimap \bot$.
[5] Recall that a minimally inconsistent set Y is an inconsistent set that does not contain inconsistent subsets.

In order to conclude, it is enough to notice that if a set of formula S is consistent (and S does not contain a tautology nor a contradiction), then, for every i, $S' = \{\mathsf{E}_i\varphi \mid \varphi \in S\}$ is also consistent. Therefore, $\bar{J}_G$ is consistent wrt the additive fragment. Thus, $cl(\bar{J}_G)$ is consistent and deductively closed wrt Linear BIAT C.

Therefore, by reasoning in Linear BIAT C about the set of formulas that are obtained by majority, i.e. about $\bar{J}_G$, we can consistently model the actions of a majoritarian group. In order to exemplify that $\bar{J}_G$ is consistent, we can show that we can infer $\mathsf{E}_{\{1,2\}\sqcup\{1,3\}}(A \otimes B)$ from formulas in $\bar{J}_G$, however this does not contradict $\mathsf{E}_{\{2,3\}} \sim (A \,\&\, B)$.

Firstly,

$$\frac{\mathsf{E}_{\{1,3\}}A \vdash \mathsf{E}_{\{1,3\}}A \text{ (as.)} \qquad \vdash \mathsf{E}_{\{1,3\}}A \multimap (\mathsf{E}_{\{1,2\}}B \multimap \mathsf{E}_{\{1,3\}}A \otimes \mathsf{E}_{\{1,2\}}B) \text{ (ax 4)}}{\mathsf{E}_{\{1,3\}}A \vdash \mathsf{E}_{\{1,2\}}B \multimap \mathsf{E}_{\{1,3\}}A \otimes \mathsf{E}_{\{1,2\}}B)} \multimap\text{-rule}$$

Then,

$$\frac{\mathsf{E}_{\{1,3\}}A \vdash \mathsf{E}_{\{1,2\}}B \multimap \mathsf{E}_{\{1,3\}}A \otimes \mathsf{E}_{\{1,2\}}B) \qquad \mathsf{E}_{\{1,2\}}B \vdash \mathsf{E}_{\{1,2\}}B \text{ as.}}{\mathsf{E}_{\{1,3\}}A, \mathsf{E}_{\{1,2\}}B \vdash \mathsf{E}_{\{1,3\}}A \otimes \mathsf{E}_{\{1,2\}}B} \multimap$$

Finally, by means of Axiom 18 and the $\multimap$-rule:

$$\frac{\mathsf{E}_{\{1,3\}}A, \mathsf{E}_{\{1,2\}}B \vdash \mathsf{E}_{\{1,3\}}A \otimes \mathsf{E}_{\{1,2\}}B \qquad \vdash \mathsf{E}_{\{1,3\}}A \otimes \mathsf{E}_{\{1,2\}}B \multimap \mathsf{E}_{\{1,3\}\sqcup\{1,2\}}(A \otimes B)}{\mathsf{E}_{\{1,3\}}A, \mathsf{E}_{\{1,2\}}B \vdash \mathsf{E}_{\{1,3\}\sqcup\{1,2\}}(A \otimes B)}$$

In order to show that $\mathsf{E}_{\{1,2\}\sqcup\{1,3\}}(A \otimes B)$ and $\mathsf{E}_{\{2,3\}} \sim (A \,\&\, B)$ are not contradictory in Linear BIAT C, we can notice that $A \otimes B$ and $\sim (A \,\&\, B)$ are *not* inconsistent in linear logic. By looking at the semantics of the two formulas, they in fact state quite different things: the former states that there are two truth-makers x_1 and x_2 one for A and one for B, whereas the latter states that the is no x such that x is both a truth-maker of A and truth-maker of B. The reason why the version of $\bar{J}_G$ in classical logic is inconsistent is that it mixes the two interpretations. Indeed, $\bar{J}_G$ turns inconsistent, if we are able to infer from it, for some combination of coalitions C, $\mathsf{E}_C(A \,\&\, B)$. But this cannot be the case in Linear BIAT C, because there is no coalition C that supports both A and B in $\bar{J}_G$. Therefore, in this setting, the discursive dilemma shows the complex nature of majoritarian reasoning, instead of being a mere logical inconsistency.

To conclude, our approach shows that it is in principle possible to talk about majoritarian group actions, provided we keep track of the complex internal structure of the alleged group agent, that is, the relationship between its internal coalitions. We may be tempted to define an agent G, the majoritarian group agent, who is responsible for all the group actions, i.e. of the formulas in $\bar{J}_G$. This can be done for instance by means of the following definition: *if* $\mathsf{E}_C\varphi$ *is in* $\bar{J}_G$*, for some* C *and* φ*, then* $\mathsf{E}_G\varphi$. The agency of G, E_G, needs to be carefully investigated because it has to reflect the complexity of the structure of the coalitions. For instance, we have to prevent axiom 17 to hold for E_G. Otherwise, we will end up facing again inconsistent outcomes: in the example above, E_GA and E_GB imply $\mathsf{E}_G(A \,\&\, B)$ and that would contradict $\mathsf{E}_G\neg(A \,\&\, B)$. By contrast, it is possible to prove that a version of axiom 18 is harmless. We need to replace the disjoint union of coalitions with an operation of composition such that it is idempotent on G: $X \bullet Y = X \sqcup Y$, for $X, Y \neq G$ and $G \bullet G = G$. By rephrasing Axiom 18 for G and $\bullet$, we have Axiom 18': $\mathsf{E}_G A \otimes \mathsf{E}_G B \multimap \mathsf{E}_{G \bullet G = G}(A \otimes B)$. The reason why axiom 18' does not lead to inconsistency is that formulas in the scope of E_G other than those introduced by axiom 18' are additive. Therefore, there is indeed a viable notion of majoritarian group agent G, the definition of its agency can be approached by means of a modality E_G that satisfies axiom 16, 18' and rules (E_G(re)) and ($\sim$ nec) of Definition 2. The actions of the majoritarian group depend on the structure of its coalition and the formulas of linear logic can express such constraints. Suppose that A and B are preconditions for the action O. We have two ways of expressing it, an additive and a multiplicative way: $\mathsf{E}_G(A \,\&\, B) \multimap \mathsf{E}_G O$ and $\mathsf{E}_G(A \otimes B) \multimap \mathsf{E}_G O$. In the former case, O is pursued by the group only if a single coalition of agents would pursue A and B; in the latter case, O is pursued even if the coalitions that support A and B are different. This means that, in the example above, if the additive constraint is assumed, then the group shall not pursue O, whereas in case the multiplicative constraint is chosen, from $\mathsf{E}_G(A \otimes B)$ and $\mathsf{E}_G(A \otimes B) \multimap \mathsf{E}_G O$, we can infer that O is performed. We leave the detailed treatment of E_G and of its further principles for future work.

6 Conclusion

We have seen that there is a viable alternative to classical logic for modelling group actions, when group attitudes are defined by majority. We have used a logic of bringing-it-about agency grounded on a propositional logic that is tailored to reflect fine-grained aspects of majoritarian reasoning. Therefore, we enabled the treatment of majoritarian groups as BDI agents, since we can show that, for any circumstances, the group guarantees a modicum of rationality. Future work concerns the study of the computational complexity of the proposed logic. For instance, the logic of agency based on intuitionistic linear logic is proved to be in PSPACE in [21]. Moreover, we plan to extend the treatment that we have proposed to represent other types of collective propositional attitudes. It is possible to provide decidable first-order versions of substructural logics in order to view preference aggregation within judgment aggregation [18]. For other types of attitude, such as beliefs, we plan to investigate the realm of substructural epistemic logics.

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