INTRODUCTION TO JUDGEMENT AGGREGATION

BASICS

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Overview

- The doctrinal paradox
- Motivations of Judgment Aggregation
- Judgment Aggregation, Preference Aggregation, and Economics.
- Judgment Aggregation in Computer Science and Multiagent Systems.
- Basics of the theory of Judgment Aggregation.

THE DOCTRINAL PARADOX: THE CASE

- The famous case of doctrinal paradox that actually emerged in the deliberative practice of the U.S. Supreme Court, namely the case of *Arizona v Fulminante* [KS93].
- This case originally motivated the study of judgment aggregation, as well as an important debate on the legitimacy of collective decisions, cf. [KS93] and [LP02a, Ott10].
- The Court had to decide whether to revise a trial on the ground of the alleged coercion of the defendant's confession. The legal *doctrine* prescribes that a trial must be revised if and only if both the the confession was coerced and the confession affected the outcome of the trial.

- At the mere level of propositional logic, we formalise the propositions involved as follows:
- p for "the confession was coerced",
- q for "the confession affected the outcome of the trial",
- r for "the trial must be revised".
- The legal doctrine is then captured by the formula of classical propositional logic
- $p \wedge q \leftrightarrow r$.
- We only report the votes of three out of the nine Justices of the Supreme Court and we label them by 1, 2, and 3. Individual votes are faithfully exemplified by the following profile.

The individuals involved in the collective decisions had this profiles of opinions (of judgments):

	p	$p \wedge q$	q	$p \wedge q \rightarrow r$	r
1	1	1	1	1	1
2	1	0	0	1	0
3	0	0	1	1	0

- What is the view of the court?
- Do we need to select a view that is blamed to be the view of the court, or we can simply agree to disagree?

- In fact, the US Supreme Court has to agree on a collective view.
- To solve the possible disagreement among its members, the doctrine impose to vote by majority:

	p	$p \wedge q$	q	$p \wedge q \to r$	r
1	1	1	1	1	1
2	1	0	0	1	0
3	0	0	1	1	0
maj.	1	0	1	1	0

- Any problem?

	p	$p \wedge q$	q	$p \wedge q \rightarrow r$	r
1	1	1	1	1	1
2	1	0	0	1	0
3	0	0	1	1	0
maj.	1	0	1	1	0

By majority we obtain:

- p is accepted (because of agent 1 and 2),
- q is accepted (because of 1 and 3),
- the legal doctrine $p \wedge q \rightarrow r$ is accepted (because it is unanimously accepted),
- r is rejected.

By viewing the rejection of r as the acceptance of $\neg r$, as usual in this setting, we can easily see that the view of the court, that is the following set

$$\{p, q, p \wedge q \rightarrow r, \neg r\}$$

is inconsistent.

WHY A *paradox*?

- The doctrinal paradox shows that, although each individual set of accepted propositions is consistent, the majority rule does not preserve consistency at the collective level.
- This resembles the Condorcet's Paradox about preferences: although individuals have rational preference (e.g. they are transitive), the collective preference obtained by majority is not (e.g. it is ciclic).
- It has been perceived as a serious threat to the actual practice of the deliberative courts.
- It is worth stressing that the doctrinal paradox emerged in the actual practice of the deliberative courts, it is not a cleverly designed example, a mathematical oddity, or a thought experiment.

WHY NOT A *dilemma*?

- List and Pettit [LP02a] argued that the doctrinal paradox exhibits a dilemma between a *premise-based* and a *conclusion-based* reading of the majoritarian aggregation.
- In the previous example suppose that:

  premises: p, q, $p \wedge q \rightarrow r$
  conclusion: r

- The premise-based reading let the individuals vote on the so called premises p and q, and $p \wedge q \rightarrow r$, and then infer the conluison Thus, r is the accepted.
- The conclusion-based reading let each individual draw the conclusions by reasoning autonomously on the premises, then it aggregates the sole conclusions. In this case, r is rejected.

Premises vs Conclusions

From the perspective of the justification of the public decisions of the Court, both the premise-based and the conclusion-based view are in principle justifiable:

- The premise-based reading appeals to the fact that individuals may collectively assess the premises and let logic (or logic and the doctrine) draw the conclusion.
  The group draws the conclusion.
- The conclusion-based reading appeals to the fact that each individual is autonomous in making his decision about the issues.
  Each individual autonomously draw her or his conclusion.

Discursive dilemmas show that this two views may produce incompatible outcomes.

Doctrinal paradox and discursive dilemma

- Nowadays, the terminology is used in the following way:
- Discursive dilemma are the general case of possible inconsistent aggregation,
- Doctrinal paradoxes refers to the original cases discussed by Kornhauser and Sager [KS93] where agents unanimously agree on the legal doctrine.
- In the case of the doctrinal paradox, one may view the legal doctrine as a constraint which is not apt for discussion among agents.

- In fact, in the doctrinal paradox, one assumes that the *doctrine* is unanimously accepted by the individuals.
- This is not necessary for getting an inconsistent outcome under the majority rule.

	p	$p \rightarrow q$	q
1:	Yes	Yes	Yes
2:	No	Yes	No
3:	Yes	No	No
maj. :	Yes	Yes	No

- The collective set is

$$\{p, p \rightarrow q, \neg q\}$$

which is again inconsitent.

Discursive dilemmas II

One can produce a discursive dilemma with conjunctions or disjunctions:

	p	$p \wedge q$	q
1:	Yes	Yes	Yes
2:	No	No	Yes
3:	Yes	No	No
maj. :	Yes	No	Yes

	p	$p \vee q$	q
1:	No	Yes	Yes
2:	Yes	Yes	No
3:	No	No	No
maj. :	No	Yes	No

A few questions about those paradoxes/dilemmas

- Are they just unfortunate oddities?
- Are they a flaw of the majority rule?
- Do they concern other voting procedures?
- Do they depend on the logical language? Are there *safe* fragments of the propositional language?
- Do they depend on the logic that grounds our reasoning?

Theory of judgement aggregation

To provide precise answers to the previous questions, List and Pettit (and others) developed the theory of JA:

- A formal setting for defining aggregation procedures.
- A formalisation of the properties of the aggregation procedure (labelled *axioms* in this setting).
- Adapt the methodology of *social choice theory*, which traditionally focuses on *preference aggregation*, to study the aggregation of logical propositions.
- *Axiomatic method*: one defines a set of axioms that express desirable properties of aggregation procedures and investigate whether this axioms are consistent, satisfiable, redundant, etc.
- This is the methodology introduced in social choice theory by Kenneth Arrow [Arr63], who was an undergraduate student of Tarski.

- After the work of Kornhauser and Sager, the doctrinal paradox was mainly studied by legal scholars, political scientists, and political philosophers (Philip Pettit).
- The work of List and Pettit provided a formalisation of the problem that allowed exporting the techniques and the observations of JA to other academic fields, in particular: Logic, AI, Knowledge Representation, Multiagent Systems.

Introductions to JA

- In Economics: List and Puppe survey paper [LP09], [LP10].
- In Computer Science: [GP14], [End16]

Judgment aggregation, the framework[1]

- Let $\mathcal{L}_{PS}$ be a propositional language with atomic symbols *PS*.
- An agenda is a finite nonempty set $\Phi \subseteq \mathcal{L}_{PS}$ not containing any doubly-negated formulas that is closed under complementation (i.e., if $\alpha \in \Phi$ then $\sim\alpha \in \Phi$).

[1] This presentation is based on Endriss, Grandi and Porello, AAMAS 2010 [EGP10], a rephrase of List and Pettit 2002.

A judgment set is any subset of the agenda Φ.
We call a *judgment set* $J \subseteq \Phi$:

- **complete** if $\alpha \in J$ or $\sim\alpha \in J$ for all $\alpha \in \Phi$;
- **complement-free** if for all $\alpha \in \Phi$ it is not the case that both α and its complement are in J;
- **consistent** if there exists an assignment that makes all formulas in J true.

- Complement-freeness is actually a merely syntactic notion. It is weak form of consistency.

AGGREGATION PROCEDURES

- Denote by $J(\Phi)$ the set of all complete consistent subsets of Φ.
- Given a finite set $N = \{1, \dots, n\}$ of $n \geqslant 3$ *individuals* (or *agents*), denote with $\mathbf{J} = (J_1, \dots, J_n)$ a *profile* of judgment sets, one for each individual.

An **aggregation procedure** for agenda Φ and a set of n individuals is a function $F : J(\Phi)^n \to \mathcal{P}(\Phi)$.

- Notice that by putting the powerset of Φ as codomain of F, we may allow for inconsistent sets of judgements.

Let $N_\varphi = \{i \in \mathcal{N} \mid \varphi \in J_i\}$.

The majority rule is defined by:

$$F(\mathbf{J}) = \{\varphi \in \Phi \mid |N_\varphi| > \frac{n+1}{2}\}$$

The class of (uniform) quota rules, for a certain threshold q, is defined by:

$$F(\mathbf{J}) = \{\varphi \in \Phi \mid |N_\varphi| > q\}$$

Conditions on the aggregation: fairness axioms

Fairness conditions:

- **Unanimity** (U): If $\varphi \in J_i$ for all i then $\varphi \in F(\mathbf{J})$.
- **Anonymity** (A): For any profile $\mathbf{J}$ and any permutation $\sigma : N \to N$ we have $F(J_1, \ldots, J_n) = F(J_{\sigma(1)}, \ldots, J_{\sigma(n)})$.
- **Neutrality** (N): For any φ, ψ in the agenda Φ and profile $\mathbf{J} \in J(\Phi)$, if for all i we have that $\varphi \in J_i \Leftrightarrow \psi \in J_i$, then $\varphi \in F(\mathbf{J}) \Leftrightarrow \psi \in F(\mathbf{J})$.
- **Independence** (I): For any φ in the agenda Φ and profiles $\mathbf{J}$ and $\mathbf{J}'$ in $J(\Phi)$, if $\varphi \in J_i \Leftrightarrow \varphi \in J'_i$ for all i, then $\varphi \in F(\mathbf{J}) \Leftrightarrow \varphi \in F(\mathbf{J}')$.
- **Systematicity** (S): For any φ, ψ in the agenda Φ and profiles $\mathbf{J}$ and $\mathbf{J}'$ in $J(\Phi)$, if $\varphi \in J_i \Leftrightarrow \psi \in J'_i$ for all i, then $\varphi \in F(\mathbf{J}) \Leftrightarrow \psi \in F(\mathbf{J}')$.
- **Monotonicity** (M^{I}): For any φ in the agenda Φ and profiles $\mathbf{J} = (J_1, \ldots, J_i, \ldots, J_n)$ and $\mathbf{J}' = (J_1, \ldots, J'_i, \ldots, J_n)$ in $J(\Phi)$, if $\varphi \notin J_i$ and $\varphi \in J'_i$, then $\varphi \in F(\mathbf{J}) \Rightarrow \varphi \in F(\mathbf{J}')$.

- Notice that systematicity is just the conjunction of independence and neutrality.
- Systematicity was originally used in [LP02b], and it has been split in the two conditions afterwards.

Conditions on the aggregation: rationality axioms

Rationality conditions:

- F is **weakly rational** iff $F(\mathbf{J})$ is *complete* and *complement-free* for every $\mathbf{J} \in J(\Phi)$.
- F is **rational** iff $F(\mathbf{J})$ is *complete* and *consistent* for every $\mathbf{J} \in J(\Phi)$.

- Notice that consistency entails complement-freeness, thus rationality entails weak rationality.
- Rationality conditions are preservation properties: for every $\mathbf{J}$ (which is assumed to be rational), $F(\mathbf{J})$ is rational as well.
- Sometimes rationality is called *collective rationality*, to stress the fact that it applies to the collective set of judgments.

In JA, one can provide the following types of results:

- Possiblity results: There exists an aggregation function that satisfies the selected axioms.
- Impossibility results: There is no aggregation function that satisfies the selected axioms.
- Characterisation results: The aggregation function f is characterised by a certain number of axioms.

An impossibility result

- The first important result in JA, proved by List and Pettit (*Economics and Philosophy*, 2002).

Theorem (List and Pettit, 2002)

(On sufficiently complex agendas) there is no rational aggregation procedure satisfying (A) and (S).

- In particular the majority rule is not rational.
- This result answers to the question concerning whether the dilemmas are peculiar of the majority rule. No, they are not. They are a much more serious threat to the legitimacy of collective decisions.
- This is the analogous of Arrow's theorem for sets of propositions (instead of preference). [Arr63]
- We shall see what "sufficiently complex agendas" means in a while. For now you can take the agenda of any discursive dilemma that we have previously introduced.
- E.g. $\{p, q, p \wedge q, \neg p, \neg q, \neg(p \wedge q)\}$

- The theorem shows that the majority rule is no rational (i.e. does not preserve completeness and consitency)
- Is it weakly rational (i.e. does it preserve completeness and complement-freenes)?

We can characterise the majority rule as follows:

Theorem (EGP, AAMAS 2010)

If the number of individuals is odd, an aggregation procedure F satisfies (A), (S) and (M) and *weak rationality* (completeness and complement-free, i.e. not consistency) if and only if F is the majority rule.

We can also characterise the class of uniform quota rules:

Theorem (EGP, AAMAS 2010)

An aggregation procedure F satisfies (A), (S) and (M) if and only if F is an uniform quota rule.

- Notice that weak-rationality fails for quota rules for arbitrary q.

We introduce the concept of safety of an agenda for a set of axioms.

Definition

An agenda Φ is *safe* for the set of axiom AX iff every aggregation procedure that satisfies all the axioms in AX is rational (i.e. complete and consistent).

[EGP12].

Median property (Mp)

We say that an agenda Φ satisfies the **median property** (MP), if every non-trivially inconsistent subset of Φ has itself an inconsistent subset of size 2.

E.g. $\{\neg p, \underbrace{p, p \rightarrow q, \neg q}, \neg q, \neg(p \rightarrow q)\}$

- At least a minimally inconsistent set of cardinality larger than three.

Theorem (List and Puppe, 2009, EGP, 2010)

An agenda Φ is safe for the majority rule if and only if Φ satisfies the MP.

- In this case safety results and possibility results coincide (the majority rule and the class of axioms that characterise it)
- Safety results for sets of aggregation procedures satisfying certain axioms are provided in [EGP12].

- Nothing binds us to use mere propositional logic. We can adapt the framework of JA to deal with many logical languages.
- In fact, JA can be construed as a theory of the aggregation of general *propositional attitudes* [DL10].
- E.g. beliefs, desires, goals, norms, preferences, ...
- An aggregative view of collective attitudes is quite general and ontologically neutral concerning the status of the collective entity to which the collective attitude is ascribed.

The focus of this introduction

- We focus here on the preservation of consistency.
- We study the notion of consistency provided by a number of logics with application in AI and MAS and we discuss Judgment Aggregation with respect to those logics.

We do not focus on.

- Studying in detail the aggregation procedure designed for a suitable task.
- Computational complexity.
- Manipulation.
- Relationship between JA and belief merging.
- Relationship between JA and preference aggregation.

OBSERVATIONS I

- The notion of consistency that is not preserved by the majority rule is the notion of consistency defined with respect to *classical logic*.
- It is therefore interesting to investigate whether there are meaningful notions of non-classical consistency that are preserved by the majority rule, cf [Por17].
- The doctrinal paradox involves a number of individual and collective propositional attitudes such as individual and collective beliefs (concerning whether the confession was coerced) obligations (e.g. the legal doctrine), and actions (e.g. the revision of the trial).
- We abstract from this aspects here (cf [Por18]).

Observations II

- The distinction between the premise and the conclusion based readings shows that in the doctrinal paradox there are inferences that are performed at the *individual* level and inferences that are performed at the *collective* level.
- In the premise-based reasoning, once p, q, and $p \wedge q \rightarrow r$ are accepted, the inference that draws r is performed only by *a minority* of individuals: indeed, this reasoning step is performed only at the collective level on the propositions that have been accepted by majority.
- It is then interesting to investigate whether it is possible to make distinction between inferences performed at the individual level and inferences performed at the collective level visible, by means of the logical modelling, cf [Por18].

- Introduction to Judgement Aggregation, foundational aspects, and the basics of the theory. ✓
- Judgment Aggregation in Description Logics, Ontology Aggregation.
- Judgment Aggregation in Non-Classical Logic: Modal Logics, Weak Logics (Linear and Relevant Logics).

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