Overview

- Judgment Aggregation in extensions of classical logic.
- An inferential view of collective rationality.
- Non-classical rationality.
- Judgment aggregation in logics that are “weaker” that classical logic

This part is based on [Por13, Por17].

We study aggregation functions

$$F : J(\Phi_L)^n \to \mathrm{P}(\Phi_L)$$

- Where L is a given logic. It is possible to rephrase the model of JA for dealing with general logics.
- We omit the details here, we refer to [Por17].

Safety of a logic

- We introduce the concept of safety of a logic for a class of aggregation procedure, in order to assess the judgment aggregation wrt. a given logic L.

Defintion (Safety of a logic)

A logic L is safe for a set of axioms AX iff for every aggregation function F (defined accordingly) and every agenda Φ_L, F is rational.

Proposition

Classical logic is not safe for the class of axioms that characterise the majority rule.

Since the majority rule is not rational for at least some agendas in classical logic (e.g. those of the discursive dilemma), classical logic is not safe for the set of axioms that characterise the majority rule.

- If X is minimally inconsistent in classical logic, then X is minimally inconsistent in any conservative extension of classical logic.
- Therefore, there is no hope to mend propositional inconsistency by enriching the language of the logic.

Proposition

Any extension of classical logic is not safe for the set of axioms that characterise the majority rule.

E.g. modal logics, first-order logic.

- We turn now to to study systems that are weaker than classical logic.
- To do that, we introduce the sequent calculus as a proof-theory of this logics.

INFERENTIAL VIEW OF LOGIC: SEQUENT CALCULUI

We approach the problem of collective rationality by using the proof-theoretic account of logic.

We use Gentzen *sequent calculus* because it allows handle reasoning in a number of different logics.

- A sequent is an expression of the form $\Gamma \vdash \Delta$ where Γ and Δ are (sets of) formulas. Formulas in Γ are *premises* and those in Δ are *conclusions*.
- The intuitive reading of the sequent is "the conjunction of formulas in Γ entails the disjunction of the formulas in Δ".

In sequent calculus for classical logic, the meaning of the conjunction is defined by means of the following rules:

$$\frac{\Gamma \vdash A \qquad \Delta \vdash B}{\Gamma, \Delta \vdash A \wedge B}\wedge \qquad \frac{\Gamma, A, B, \Gamma' \vdash \Delta}{\Gamma, A \wedge B, \Gamma' \vdash \Delta}\wedge$$

STRUCTURAL RULES AND REASONING

The structural rules of *contraction* and *weakening*:

$$\frac{\Gamma, A, A, \vdash \Delta}{\Gamma, A \vdash \Delta}\,C \qquad \frac{\Gamma \vdash \Delta, A, A}{\Gamma \vdash \Delta, A}\,C$$

Multiple occurrences are irrelevant

$$\frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta}\,W \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, A}\,W$$

Monotonicity of the entailment

Structural rules determine the behavior of logical connectives: they make the following two presentations of the rules for conjunction equivalent:

$$\frac{\Gamma \vdash A \qquad \Delta \vdash B}{\Gamma, \Delta \vdash A \wedge B}\,\wedge \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \wedge B}\,\wedge$$

(*multiplicative and additive presentation*)

- By rejecting structural rules, there are two conjunctions (disjunctions) with different logical behaviors.

Linear logic

- (Girard, 1987). Linear logic rejects the global validity of contraction and weakening.
- By rejecting contraction and weakening, there is no longer a single conjunction (disjunction), we are lead to admit two: $\otimes$ "tensor" and & "with":

$$\frac{\Gamma \vdash A \qquad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}\,\otimes \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \,\&\, B}\,\&$$

(*multiplicative and additive conjunction*)

Discursive dilemmas: inferring the contradiction

	a	$a \wedge b$	b	$\neg a$	$\neg(a \wedge b)$	$\neg b$
i_1	1	1	1	0	0	0
i_2	1	0	0	0	1	1
i_3	0	0	1	1	1	0
maj.	1	0	1	0	1	0

- The fact that $\{a, b, \neg(a \wedge b)\}$ is not consistent means (proof-thoeretically) that we can infer a contradiction from a, b, $\neg(a \wedge b)$.

Inferring the collective contradiction

- We infer the contradiction by reasoning in classical logic as follows.
- Suppose that the (winning) coalition of agents that support an elected formula acts as the premise of a sequent that entails the formula, that is the winning coalition is what makes the formula true at the collective level.

$$\dfrac{\dfrac{\dfrac{}{\{i_1, i_2\} \vdash a}\text{ majority} \qquad \dfrac{}{\{i_1, i_3\} \vdash b}\text{ majority}}{\{i_1, i_2\}, \{i_1, i_3\} \vdash a \wedge b}\,R\wedge \qquad \dfrac{}{\{i_2, i_3\} \vdash \neg(a \wedge b)}\text{ majority}}{\{i_1, i_2\}, \{i_1, i_3\}, \{i_2, i_3\} \vdash (a \wedge b) \wedge \neg(a \wedge b)}\,R\wedge$$

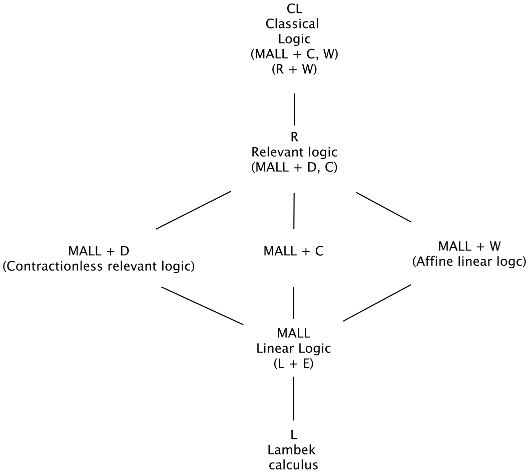
- By dropping W and C, can we still infer the contradiction? That is, is the group still inconsistent wrt LL reasoning?
- If the group reasons in linear logic, the non-logical axioms are again: $\{i_1, i_2\} \vdash a$, $\{i_1, i_3\} \vdash b$ and $\{i_2, i_3\} \vdash \neg(a \wedge b)$.

$$\dfrac{\dfrac{\{i_1, i_2\} \vdash a \qquad \{i_1, i_3\} \vdash b}{\{i_1, i_2\}, \{i_1, i_3\} \vdash a \otimes b}\, R\otimes \qquad \{i_2, i_3\} \vdash \neg(a \wedge b)}{\cdots}$$

- The group can infer $a \otimes b$ by using two different coalitions. What is the interpretation of $\neg(a \wedge b)$ in linear logic?

- If $\neg(a \wedge b)$ is interpreted as the multiplicative conjunction, then we have inferred again a contradiction.
- However, if $\neg(a \wedge b)$ is interpreted additively as $\neg(a \,\&\, b)$, then $a \otimes b$ and $\neg(a \,\&\, b)$ are *not* inconsistent in linear logic!
- $a \otimes b, \neg(a \,\&\, b) \nvdash_{LL} \emptyset$ and $a \,\&\, b$ cannot be inferred in the previous case.

A MAP OF SUBSTRUCTURAL LOGICS



The lack of weakening permits that a consistent set X may have inconsistent subsets, which may violate complement-freeness, even in the case of consistency of X.

Thus, for substructural non-monotonic logic *consitency* is replaced by *robust consistency*:

Definition

We say that a set (multiset, list) J is *robustly consistent* if J is consistent and every proper subset (submutliset, sublist) J' of J is.

Theorem

For every agenda defined in Additive Linear Logic, the majority rule is collectively rational wrt Additive Linear Logic. I.e. Additive linear logic is safe for the set of axioms that characterise the majority rule.

The proof is based on the following points.

- An agenda is *safe* for an aggregation procedure F if there is no profile defined on it such that F that violates collective rationality,
- An agenda is safe for the majority rule iff every minimally inconsistent subset of the agenda has cardinality at most 2.
- In additive linear logic, every provable sequent contains at most two formulas.

EXTENDING THE POSSIBILITY RESULT

Theorem

For every agenda defined in Additive Linear Logic plus contraction (C) and distributivity (D) (that is, the additive fragment of relevant logic), the majority rule is collectively rational wrt Additive Linear Logic. I.e. Additive linear logic plus (C) and (D) is safe for the set of axioms that characterise the majority rule.

- Notice that by adding W or by adding the multiplicative conjunction $\otimes$ or $\parr$ the safety result is lost.

	MAJ (WR, A, I, N, M)	Quota rules (A, I, N, M)
Φ_{CL}	safe iff MP	safe iff kMP
Φ_{IL}	safe iff MP	safe iff kMP
Φ_{L}	safe iff MP	safe iff kMP
Φ_{MALL}	safe iff MP	safe iff kMP
Φ_{MLL}	safe iff MP	safe iff kMP
Φ_{ALL}	always safe	safe with $m \geqslant n/2$
$\Phi_{\mathrm{ALL}W}$	safe iff MP	safe iff kMP
$\Phi_{\mathrm{ALL}C}$	always safe	safe with $m \geqslant n/2$
Φ_{R}	safe iff MP	safe iff kMP
Φ_{AR}	always safe	safe with $m \geqslant n/2$

TABLE: Summary of results concerning the safety of agendas and logics for sets of axioms.

References I

Daniele Porello.
A proof-theoretical view of collective rationality.
In *IJCAI 2013, Proceedings of the 23rd International Joint Conference on Artificial Intelligence, Beijing, China, August 3-9, 2013*, 2013.

Daniele Porello.
Judgement aggregation in non-classical logics.
Journal of Applied Non-Classical Logics, 27(1-2):106–139, 2017.