

A note on the representation of relations in conceptual spaces

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Cognitive semantics has been proposed by Gärdenfors (2000) as an alternative to truth-conditional semantics that models the relationship between language and mental representations of cognitive agents. Similar motivations emerged in philosophical logic, since a number of non-classical logics have been interpreted as modelling the reasoning capability of a knowing subject. In particular, *relevant logics* have been discussed as logics of information in (Mares, 2010). In a recent paper, Masolo and Porello (2015) tightened the connection between the tradition of cognitive semantics and relevant logics, by providing a model of a propositional relevant logic in terms of conceptual spaces. There, the interpretation is restricted to propositions that correspond to predications of *properties*. This restriction is motivated by the fact that, besides a recent treatment of the part-whole relation (Fiorini *et al.*, 2014), a precise and general method to represent *n*-ary *relations* in conceptual spaces is—as we will try to better motivate in the rest of the paper—still missing. In our view, this is also linked to the fact that the aim of conceptual spaces is to rep-

resent similarity relations between single objects. While, “[a]s accounts of similarity relations between what in the literature are sometimes called “multipart scenes” (which typically involve multiple objects or figures), [conceptual spaces] may fare less well” (Deacock and Douven 2011, p.67).

Discussing the conceptual content of relations is then important both to provide a first-order model of relevant logics that fully justifies relevant logics as logics of a cognitively situated agent and to better understand what are the main problems linked to the representation of relations within conceptual spaces. In the remainder of this note, we will present a number of preliminary observations on the latter topic.

We start by recalling the philosophical distinction between *internal* and *external* relations as it will suggest a crucial point about relations in conceptual spaces. Without entering the details of a complex debate, we simply present the distinction as it has been formulated by Russell (1992). Roughly, a relation R is *internal* if the truth-value of aRb is inferable from some facts about a and b only. By contrast, *external relation* requires additional information in order to assess the truth-value of aRb . There are two well established positions concerning the status of external relation: Reductionists maintain that there is no relational fact that is a truth-maker of a relational sentence, they can always be reduced to monadic facts (e.g. Parsons, 2009); Anti-reductionists state that there exist relational sentences that require irreducible relational facts as truth-makers (e.g. Russell, 1992, Armstrong, 1997).

Gärdenfors discusses very briefly how relations can be represented within the framework of conceptual spaces. However, he seems to embrace a reductionist position: “[a] relation between two objects can be seen as a simple case of a *pattern* of the location of the objects along a particular quality dimension” (Gärdenfors 2000, p.93). While objects are represented by points and properties (concepts) by (sets of) convex regions, relations are represented by *higher level properties*, properties of tuples of objects in a *product space* (indeed, a quite mathematical approach). In his view, *product spaces* reduce to Cartesian products $X_1 \times \dots \times X_n$ whose metric is a *function* of the metrics of the X_i components. In general, this reduction holds for conceptual spaces that explain higher level similarities and categorizations by reducing them to the ones along the quality dimensions. Actually, this position is anchored to *Multidimensional Scaling* (MDS)—to which Gärdenfors commits at least for the identification of quality domains—whose main idea is to represent (quantitative) similarity judgments as distances between points of an n -dimensional space that are a function of, are *reducible* to, the distances in the projection-spaces.

Therefore, the relations introduced by Gärdenfors may be considered as *internal*, in the sense that the truth-value of aRb depends only on similarity

judgments along quality dimensions concerning *a* and *b* only. From a more ontological perspective, Gärdenfors seems to suggest that similarities between *configurations* of the world can be always explained in terms of similarities between the involved *objects*, hence our claim about his reductionist stance concerning external relations.

Nevertheless, this reductionist stance is not shared by the whole community working on conceptual spaces. For instance, Aisbett and Gibbon (2001) and Fiorini (2014) seem to adopt a weaker notion of product space. Product spaces are characterized in terms of their projections into the components that must be continuous morphisms that preserve the betweenness. In this case, the metric of a product space is characterized in terms, *but is not necessarily a function of*, the metric of its components, i.e., the similarity judgments concerning the product space must be separated from the ones concerning the components. From an epistemological or scientific viewpoint, Borg and Groenen (2005) individuate several purposes of MDS, e.g., to test criteria and theories that explain judgments of similarity or to discover the dimensions that underlie judgments of similarity. In a non-reductionist perspective, conceptual spaces could be used also to represent, test, and discover simple *correlations* (instead of reductions) between different kinds of similarity judgments, in the case of external relations but also, more generally, in the case of complex concepts. For instance, Fiorini *et al.* (2014) and Fiorini (2014) consider structural aspects in the scope of part-whole relations, i.e., how wholes are structured in parts. As in the case of relations, in the philosophical literature there exist reductionist and anti-reductionist positions. For instance, Koslicki (2008) claims that wholes are reducible to *mereological sums* of objects and *form* (to be introduced in the domain) while Baker (2007) assumes a *constitution-view* where the whole depends on, but is not reducible to, the constituents and the way they are configured. Actually, the whole may have properties that are not linked to the ones of its constituent as in the case of the price of a piece of art. Fiorini solves this problem by assuming that wholes are represented by the product of a structural and a holistic space, where the last one represent properties of the whole that are not necessarily reducible to the ones of the parts. This solution seems to us *ad hoc*. A non-reductionist use of conceptual spaces would facilitate the representation of simple correlations, when they exist, between whole- and constituent-similarities without the necessity of introducing the product of a holistic and structural space.

This non-reductionist perspective, in the case of product spaces, would require to accept similarities between *tuples* of objects. Goldstone and Son (2005) review some kinds of comparisons involving multiple objects where the *arrangement (structure)* of the objects influences judgments of similarity between configurations and also between single objects, i.e., the way the ob-

jects are linked to the surrounding ones impacts similarity between objects. Ontologically, tuples have a quite vacuous nature. We prefer here to refer to *states of affairs, facts, situations, or configurations*. The recognition of this kind of entities in the inventory of the world provides the base for a direct interpretation of the points of product spaces. The *structure* may then be intended as a simple property of these entities. In addition, this move allows us to manage an abstract notion of configuration that does not reduce only to the spatial one. The ‘glue’ between the objects involved in a state of affairs could be abstract enough to represent, in a unified way, external spatial relations and external relations like *loves* (psychological), *married to* (social), or *owns* (legal).

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