LOGIC IN SOCIAL CHOICES

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Abstract

This paper presents a number research topics at the interface of welfare economics, social choice theory, logic, and artificial intelligence. We will discuss in particular the role of logic in modelling the relationship between individual reasoning and group or social reasoning.

1. Introduction

In recent years, several connections have been established between mathematical logic, welfare economics and social choice theory, due to an interesting overlapping of problems and issues.

Welfare economics is concerned with the mathematical investigation of the fair distribution of resources among individuals endowed with their own preferences and points of view. Social choice theory, in a broad sense, is a branch of welfare economics that studies the aggregation of individual attitudes – beliefs, preferences, judgements, or plans – into a collective or social attitude. The aggregation of individuals' perspectives is supposed to balance between certain fairness desiderata and certain efficiency requirement. A representative branch of social choice theory is given by voting theory that studies the formal properties of voting procedures, e.g. the majority rule. For example, fairness conditions on voting procedures should impose not to favour any particular individual, while efficiency conditions aim to guarantee a rational voting outcome for every possible election.

The seminal result in social choice theory, namely Arrow's impossibility theorem, surprisingly shows that fairness and rationality are mutually incompatible in several situations.

From the point of view of formal reasoning, we could view social choice theory as discussing how individual and collective rationality can be connected by means of procedures of aggregation that fairly weight individuals' contributions.

The study of collective or group rational interaction has recently become a central issue also in mathematical logic. We observe a sort of interactive turn in logic: the object of interest is shifting from modelling individual rationality towards modelling communities of reasoners that share information and reasoning skills. This shift can be seen in the growing interest in the logical focus on game theory and multiagent systems (van Benthem, 2011).

The connection between social choice theory and computer science is particularly fruitful for both fields, allowing for exchanging techniques and ideas in both directions. For example, the study of the computational complexity of deciding properties of voting procedures is a new important criterion for evaluating them. Relevant questions are, for example: “how hard is to compute the winner”, “how hard would it be for an agent to manipulate the outcome of the election?”. Complexity issues are also relevant for preference elicitation. Given a certain voting rule, we may ask how hard is to retrieve the information required by the input of the procedure. For example, certain procedures require individuals to indicate just their most-preferred candidate, others are more demanding, as they need a complete ordering of all candidates or a score to be associated with each candidate.

On the other hand, several fields in computer science (artificial intelligence (AI), multiagent systems, and knowledge representation (KR) in particular) benefit from social choice theory as a source for ideas, principles, and techniques to model the interconnections among different artificial agents.

The overlapping of problems and techniques between social choice theory, logic, and computer science constitutes a research area within the AI community, now labelled *computational social choice* (see Chevaleyre *et al.*, 2007, Endriss, 2011). Logic is an important modelling tool in this interdisciplinary field, as various approaches rely on it, for example, for designing formal languages and reasoning systems for preferences representation.

The new problems that emerge in collective or social reasoning suggest new developments of logical frameworks in order to account for the rationality related phenomena that emerge from social interaction.

In this paper, I will focus on the role of logic in modelling individual and collective reasoning for the case of the aggregation of individual opinions and preferences. In particular, we select and discuss a number of topics at the interface of logic, welfare economics and social choice theory (see Note 1).

The aim of this presentation is to highlight some formal approaches that might be of interest for cognitive science.

The remainder of this paper is organised as follows. In Section 2, we will see a succinct overview of some fundamental results in social choice theory. In particular, we introduce Arrow's theorem and some of the solutions that have been proposed in the literature in order to escape the impossibility stated by Arrow. Section 3 presents some application of logic in the recent field of *judgement aggregation*. Section 4 describes the application of logic in the field of resource allocation as a powerful tool for representing preferences, allocations of goods, deals, and also rationality constraints. Section 5 discusses some aspects of the role of logic in modelling general concepts of social choice theory.

2. **Topics in Social Choice Theory**

Although the discussion of voting rules is ubiquitous in the history of political studies, what we can assume that social choice theory in its mathematical formulation was born with the fundamental work of Kenneth Arrow. This is due not just to the impact of his impossibility theorem, that shows the inconsistency between the fairness conditions on the aggregation procedure and the rationality constraints on preferences, but also for the method and the mathematical framework that he developed.

We present the content of Arrow's theorem starting with a well-known paradox of voting. Consider three individuals, labelled 1, 2, and 3, who order the candidates (*a, b* and *c*) for an election according to the following table:

$$1: a > b > c$$
$$2: b > c > a$$
$$3: c > a > b$$

The first voter prefers a over b and b over c. A common assumption on preference ordering of individuals is that they should be *complete* (given two candidates, voters know how to rank them: $a > b$ or $b < a$); preference should be *irreflexive* (for every candidate a, not $a > a$); moreover preference should be *transitive*: if a voter prefers a over b and b over c, then she should also prefer a over c. The ideal of instrumental rationality in this framework is then defined by such properties.

We can ask how to obtain a collective ordering from the individual preferences. We are looking for procedures that reflect the individuals' views in some suitable way, namely, a rule to move from the preference orderings of agents to the preference of the society.

One possibility that seems to meet our desiderata is to vote by majority. If we compare pairs of candidates by majority, we may obtain the following order: a is preferred over b, since 1 and 3 vote this way, b is preferred over c, according to 1 and 2, and c is preferred over a, according to 2 and 3. However, from $a > b$ and $b > c$, by transitivity one has also $a > c$. Therefore, the collective ordering is not consistent with irreflexivity: namely from $a > c$ and $c > a$, one has $a > a$. Thus, even if we assume rationality of individuals, majority returns an irrational collective preference. Moreover, the social outcome that we would obtain (namely, $a > b > c > a$) is cyclic: even if individual preferences are clear and have a single most-preferred candidate, the social outcome is still undetermined.

This is the well-known *Condorcet's paradox* of pairwise majority voting. It shows a genuine problem, since majority voting is intuitively a fair voting procedure to elect collective preferences.

The impossibility theorem proved by Kenneth Arrow can be considered a generalisation of situations like Condorcet's paradox to every aggregation procedure that satisfies certain conditions that express our fairness intuitions. In order to prove this result, Arrow provides a formalisation of the notion of preference as a particular relation that satisfies the properties we described above. An important aspect is that Arrow's formalisation provides a mathematical understanding of what is a fair aggregation procedure. An aggregation procedure is a function, a *social welfare function*, that takes as input profiles of individual preferences and produces a (single) social preference. Arrow's conditions (Arrow, 1961) are then the following.

The first axiom, called *universal domain* (UD), states that the aggregation procedure is indeed a total function, it returns a social order for every possible profiles of preferences. This means that we are not excluding any possible preference ordering. Thus assumption reflects the utilitarian view that individuals are free to choose among any possible preference their ends.

The *weak Pareto property* (P), or unanimity, states that if in a profile every individual votes for *a* over *b*, then the social ordering should rank *a* over *b*.

The *independence of irrelevant alternatives* (I) states that the social preference of *a* over *b* should depend exclusively on the individual rankings of *a* and *b*, and not on the relationship that *a* and *b* might have with another (irrelevant) option *c.* (Note 2)

The *non-dictatorship* (D) condition states that the social welfare functions should not produce a preference that is always equal to the preference of a particular individual (the dictator).

Under such hypotheses, Arrow's impossibility theorem can be stated as follows:

Every social welfare function satisfying (UD), (P) and (I) and always providing rational outputs is a dictatorship.

The theorem shows that our intuitions concerning fairness and rationality are mutually inconsistent. Thus, in order to define an aggregation procedure, we should give up some of the desiderata that we saw. Alternatively, we can say that the individual standard of reasoning about preferences cannot be extended to social reasoning (by means of a fair procedure).

We cannot discuss here the role of Arrow's work on the debate of the possibility of social choice, connected in particular with the normative theory of democracy. We simply mention some directions that have been investigated in order to escape Arrow's impossibility.

A way out, that was proposed by Duncan Black (1958), is to restrict the possible profiles of preference to those that satisfy a particular condition, the *single-peakedness* condition. The intuitive meaning of this property is that individuals might share a common *dimension* of voting, for example they might agree to rank candidates on the left-right (political) spectrum.

Under this assumption, Duncan Black proved that majority rule always produces a rational outcome. The importance of this property is the following: if individuals are able to reach an agreement on what is the relevant dimension for choosing, then they can consistently use majority to aggregate their preferences.

We conclude this section by mentioning that, in order circumvent the inconsistency we saw, one way is to depart from the ordinal model of preference of Arrow's setting (Roemer, 1996). We just mention two possible departures, as we will present some application of logic in those domains.

The first one is a recent approach, directly connected to logic, labelled *judgement aggregation.* It describes the aggregation of logically connected propositions that individuals might submit for a possible election. The idea is that if individuals discuss and reason about their preferences they might reach an agreement on the relevant dimension for a given choice and thus circumvent Arrow's theorem.

The second framework is to move to welfare economics; it assumes that individual preferences are adequately described in a cardinal way, that is, they are represented by means of utility functions. The idea is that by having a more informative representation of individual preferences, that for example takes into account the intensity of their desires, Arrow's theorem does not apply.

3. **Logic and Judgement Aggregation**

In the literature on legal theory, scholars detected a deliberative *impasse,* known as *discursive dilemma*, which shares some similarities with Condorcet's paradox (Kornhauser, 1986). A discursive dilemma can be stated as follows. Suppose there are three individuals (1, 2 and 3), and there are propositions (in propositional logic) A, B and their logical conjunction A & B. Propositions may represent possible policy to elect. Suppose individuals judge the propositions involved in the following way.

As the table shows, each individual has a coherent set of judgements. However, if we aggregate the three sets by majority, we would obtain that A

is collectively judged to be true, B is collectively true, but A & B is judged to be false. Thus, accepting A and B and rejecting A & B is inconsistent. Generalizing this type of configurations, List and Pettit proved an

	A	B	A & B
1	TRUE	TRUE	TRUE
2	FALSE	TRUE	FALSE
3	TRUE	FALSE	FALSE

impossibility theorem on the aggregation of sets of judgements (List &Pettit, 2002). The intuitive meaning of the theorem resembles Arrow's result: we cannot satisfy both rationality constraints, here expressed by the rules of logic, and fairness constraints on the procedure.

In the last decade, judgement aggregation has become an interesting topic for logicians. For example, there are connections in particular with the area of *belief-merging*, see Pigozzi (2006).

Our contribution to judgment aggregation can be summarized as follows: Firstly, we developed the investigation of complexity issues in judgement aggregation. Secondly, we investigated the relationship between preference and judgement aggregation. Lastly, we discussed the models of group rationality in judgement aggregation.

3.1. *Reasoning in Judgement Aggregation*

Computational complexity plays an important role in the discussion of voting procedures. In (Endriss *et al.*, 2010), we developed a framework to discuss complexity problems in judgement aggregation. In particular, we focused on the problem of deciding whether an agenda of logically connected propositions is *safe* for a certain aggregation rule, namely, whether it guarantees consistent outcomes.

We adapted the characterisation results known from the literature (see List & Puppe, 2009) in order to set up the precise decision problem concerning the safety of an agenda. The characterisation results known in the literature state that an agenda may produce a paradox for a certain voting procedure if and only if the agenda has some particular logical connections (i.e. all the minimally inconsistent sets included in the agenda have cardinality strictly greater than two).

We proved that, in order to be safe for entire classes of aggregation rules, an agenda has to show very poor logical connections. Moreover, we proved that the problem of deciding whether a given agenda is safe for a class of aggregation procedures is highly intractable (see Note 3).

The extremely poor logical structure of the agenda required to save collective reasoning from inconsistency points at possible revisions of the classical notion of rationality. Namely, if collective reasoning is not consistent even in the case of poor logical reasoning, we can think of revising the notion of rationality implicit in (classical) logic.

This strategy has been proposed in a recent paper (Porello, 2012) that explores the possibility of modelling group reasoning by means of a constructive logic, namely linear logic, instead of classical logic.

3.2. *Preference and Judgement Aggregation*

The relationship between Arrow's result and the impossibility theorem concerning judgements has been deeply discussed in the literature. For a survey of judgement aggregation we refer to (List & Puppe, 2009), for a formal comparison with Arrow's theorem see (List & Dietrich, 2007). Besides the formal comparison of the two frameworks, it is of interest to investigate the actual relationship between the two paradoxes: Condorcet's paradox and the discursive dilemma.

We investigated this relationship between preferences and judgements from the following perspective. The basic idea is that judgements or opinions are used by individuals as justifications for their preferences. In particular, propositions expressed by individuals can be used to define *verbalisations* of the notion of dimension of the Black single-peakedness condition. The problem with the notion of dimension in the condition used by Black is that it is not clear how agents should refer to such a shared ranking of the candidates. One must assume that such a ranking already exists and is known by voters. There are cases in which this assumption is clear, e.g. the left-right order of political parties. However, what happens in general?

The model we are discussing (Porello 2009, 2012) assumes that agents can express their dimensions as judgements in a logical language. The logical propositions provide the verbalization of the (possibly) underlying dimension. Judgements represent what voters think about the possibly shared ranking.

One can prove that even if a profile produces a rational outcome on preferences, voters have to face a discursive dilemma if they try to aggregate their justifications of preferences. Even if preference aggregation is safe, on the profile of judgements that are used to verbalize the shared dimension, a discursive dilemma may emerge.

This technical result can be considered the formal version of a phenomenon detected, to my knowledge for the first time, by Ottonelli (2010) and analysed with respect to the effect of discursive dilemmas on the deliberative theory of democracy proposed by Pettit.

Further work in this direction includes the investigation of a formal model of deliberation, which shall require an investigation on the use of judgements as justifications that are publicly endorsed by agents. This research line requires a conceptual analysis of the formal properties of the meaning theory of sentences used in a public assembly. These problems are connected with recent approaches in philosophy of language and semantics that are close to pragmatism.

An analysis of how agents that discuss and reason in a deliberative situation have to align their justifications and their beliefs in order to agree on a shared dimension of voting has been proposed in (Ottonelli and Porello, 2012).

4. Logic for Resource Allocation

Logical languages for representing preferences have been developed by several authors, for example (Lang, 2004), (Boutilier & Hoos, 2001), and (Uckelman *et al.,* 2008) are the most direct reference for our approach.

The general aim of our work is to show how a non-classical logic, i.e. *linear logic,* is particularly suitable for representing and modelling economic reasoning. Linear logic has been developed by Girard (1987) and it provides a resource-sensitive account of proofs by showing precisely which assumptions are needed in the deduction. Its applications were successful, for example, in modelling processes where one could observe (computational) resource consumption. In particular, linear logic can express resource-bounded reasoning by means of the implications "if *A*, then *B*". For example, "if I have one Euro, I buy a coffee" can be modelled in a way that the resource used to get the coffee (the antecedent "I have one Euro") is actually consumed, and is not available to satisfy other conditionals in the same chain of reasoning.

By using linear logic, we developed two logical frameworks for modelling two important models of resource allocation: combinatorial auctions and distributed negotiation.

Combinatorial auctions can be described as a multiagent resource allocation problem where an auctioneer sells bundles of goods to a set of bidders. The difference with a classical auction is that here we allow bidders to express their preferences over logical combinations of goods. For example, agents can utter sentences of the form "I would pay 10 Euro to get a pair of shoes *or* a pair of gloves". Due to the richer expressive power of this framework, usually the problem of determining which is the best allocation, the allocation of goods to bidders that produces the highest revenue for the auctioneer, is computationally hard. For the state of the art of combinatorial auctions, we refer to the collection edited by Cramton, Shoham, and Steinberg (2006).

In our model (Porello & Endriss, 2010), we developed a bidding language based on linear logic formula for combinatorial auctions in which there might be different indistinguishable copies of a same good (multi-unit combinatorial auctions).

Bidders' preferences can be modelled by using linear logic conditionals. For example “if I get B, I pay n Euros”, that behaves as expected: if such a formula is satisfied, by a formula B, then it will provide (it will prove) a formula corresponding to the price n.

An interesting feature that is peculiar to our model is that we can also model *procedures* of allocation within the logical framework. Basically, we proved that we can set a correspondence between allocations of goods and proofs of particular formulas in linear logic.

Starting form this model for combinatorial auctions, we developed a model for *distributed negotiation* (Porello & Endriss, 2010). One can consider auctions as a centralised system of allocation in which one special agent, the auctioneer, decides the allocation. In distributed negotiations, several agents make deals in order to exchange their goods. Deals are accepted according to some rationality constraints. For example, a utilitarian acceptance criterion says that an agent is willing to accept a deal only if its utility increases. In our model, besides using linear logic to model agents preferences as we did in the previous work, we could also define a logical language that expresses deals and rationality constraints on deals. Again, allocations of goods are interpreted as proofs of particular formulas, and so they can be accounted for within the system in a harmonious way.

The two articles that we have just mentioned provide several elements to model economic reasoning. In particular, we motivated in those scenarios the use of a resource-bounded reasoning captured by linear logic inferences.

5. Conclusion

The aim of this paper was to present a number of problems and of formal frameworks defined in logic and artificial intelligence that might provide useful tools for the analysis of the relationship between individual and collective reasoning concerning possible decisions to make.

Logic can be used to address problems of social choice theory and it leads to a principled account of languages for describing preferences and judgments. Therefore, we can investigate qualitative properties of such languages, such as their expressive power, complexity, succinctness of the representations. Moreover, the logical representation is often an aid for studying the computational complexity of the problems at issue.

Finally, a logical framework provides the required awareness in order to specify and unveil the hypothesis of a certain view of reasoning.

Besides the technical advantages that formal languages provide, logic, which was traditionally concerned with the normative approach to an idealised reasoner, in recent years has widened its scope to account for bounded rationality or to describe features of communities of reasoners. With this respect, mathematical logic is in touch with topics discussed in cognitive science.

We have mentioned situations and results showing that classical logic reasoning cannot be used to model adequately the relationship between individual and collective reasoning and we stressed how a non-classical logics, e.g. linear logic, may provide powerful tools to model abstract agents that reason and negotiate.

The relationship between individual and collective attitudes and reasoning has been described here by means of social choice theory and welfare economics. The fairness conditions that we have mentioned can also be viewed as conditions that capture a complex notion of collective information. For example, they exclude notions of collective reasoning that simply copies a specific individuals' attitude (e.g. non-dictatorship).

The line of research at the interface of logic, artificial intelligence and welfare economics that we have presented provides then useful formal frameworks to discuss empirical modelling of and collective aspects of cognition.

Note 1. An exhaustive overview of the area is outside of the scope of this short paper. Our aim is to select a number of contributions, that invove the present author, that focus on the relationship between individual and collective reasoning.

Note 2. This condition is less intuitively clear. It leads to a non-manipulability condition: if the outcome over a and b does not change whatever c is in the set of candidates, then it is not possible to manipulate the outcome of an election introducing new alternatives.

Note 3. In particular, we proved that the problem is at the second level of the polynomial hierarchy. For all the details, see (Endriss et. al., 2010).

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